9. Multiphase Flow

Hold Up

Flow Regimes

Pressure Drop

Typical Subsea Development

Flow assurance will involve the design and operation of;

Single phase incompressible flow –

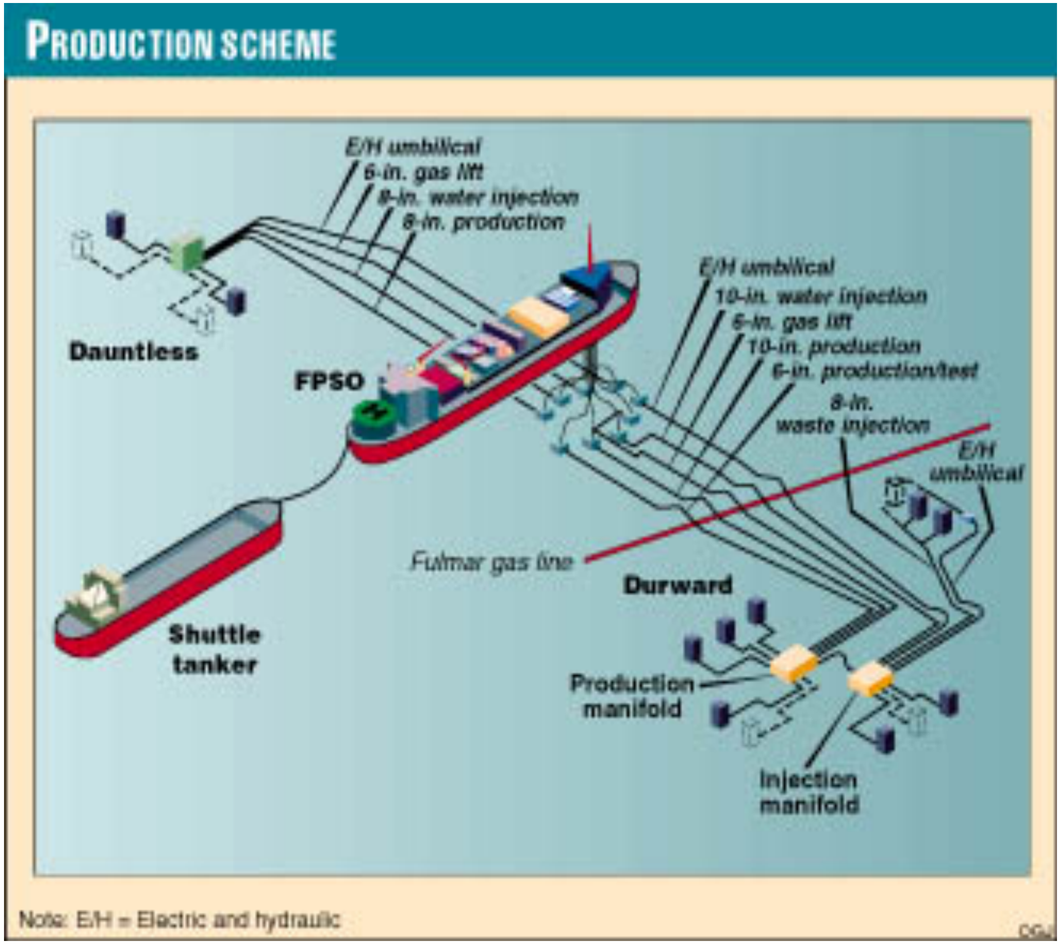
Water injection, oil export, chemical injection (methanol, corrosion inhibition etc.)

Single phase compressible flow –

Gas lift, gas export.

Multi-phase flow –

Production, test pipelines



Fluid Flow Energy Equations

- For isothermal and adiabatic flow the First and Second Laws of Thermodynamics may be combined to write (Bernoulli's Theorem):

$$\underbrace{\int \frac{dP}{\rho}}_{\text{Pressure change}} + \underbrace{g.\Delta X}_{\text{Elevation term}} + \underbrace{\frac{\Delta(v)^2}{2}}_{\text{Velocity term}} = \underbrace{-W_f}_{\text{Friction term}} \underbrace{-W}_{\text{Work term}}$$

- Darcy's Formula:

$$\Delta P_f = \frac{\rho \cdot f \cdot L \cdot v^2}{2 \cdot D}$$

Meaning for multi-phase flow

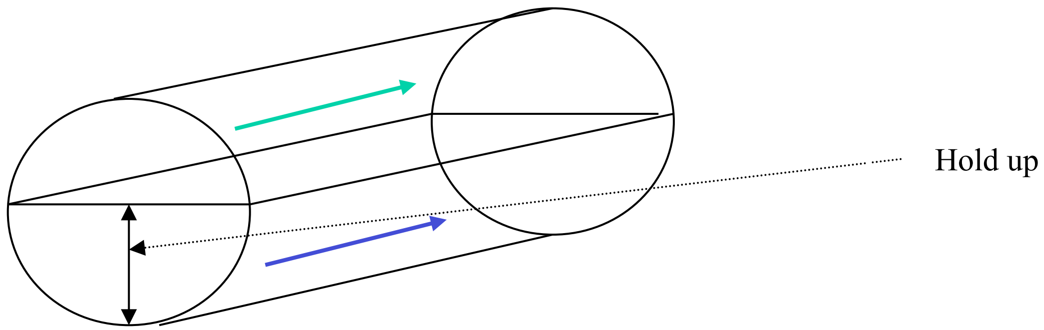
V	Volume of fluid (m^3)
P	Pressure of fluid (Pa)
ΔX	Change in elevation of fluid (m)
Δv	Change in velocity of fluid (m/s)
W_f	Work lost due to friction (kJ)
W	Work done by system (kJ)
g	Gravitational acceleration (m/s^2)
g_c	Mass-force conversion constant (kg^{-1})
f	(Moody) friction factor (-)
L	Pipe length (m)
v	Fluid velocity (m/s)
D	Pipe internal diameter (m)
ΔP_f	Frictional pressure loss (Pa)
ρ	Fluid density (kg/m^3)

<u>Note that there are two different friction factors:</u>

- Moody friction factor
- Fanning friction factor

$$f_{(Moody)} = 4 \cdot f_{Fanning}$$

Hold-Up



Two-Phase Flow

Liquid Holdup H_l

Ratio of the volume of a pipe segment occupied by liquid to the volume of the pipe segment

H_l = volume of liquid in a pipe segment/volume of pipe segment

Value varies from 0 to 1

Gas Holdup or gas void fraction H_g

$$H_g = 1.0 - H_l$$

No-Slip Liquid Holdup – λ_l

Ratio of volume of liquid in a pipe segment divided by volume of pipe segment which would exist if the gas and liquid travelled at the same velocity (no-slippage)

$$\lambda_l = q_l / (q_l + q_g)$$

Where q_l and q_g are liquid and gas volumetric flow rates

No-Slip Gas Holdup – λ_g

$$\lambda_g = 1.0 - \lambda_l = q_g / (q_l + q_g)$$

Two-Phase Velocity

The velocities of the gas and liquid in the pipe are prime variables in the prediction of the behavior of the multiphase mixture. Most multiphase flow prediction methods use the superficial gas and liquid velocities as correlating parameters. The superficial velocities are defined as the in situ volumetric flowrate of that phase divided by the total pipe cross sectional area. Or, the velocity in the pipe if the gas or liquid were flowing alone.

Mixture velocity,

$$v_m = v_{sl} + v_{sg}$$

$v_{sl} = q_l / A$ – superficial liquid velocity $\quad$ $v_l = q_l / (A.H_l)$ – actual liquid velocity

$v_{sg} = q_g / A$ – superficial gas velocity $\quad$ $v_g = q_g / (A.H_g)$ – actual gas velocity

$\lambda_l = q_l / (q_l + q_g)$, but $v_{sl} = q_l / A$ and $v_{sg} = q_g / A$, so

$$\lambda_l = v_{sl} / (v_{sl} + v_{sg}) = v_{sl} / v_m$$

Some researchers refer to superficial velocity as j, the volumetric flux.

Horizontal Flow Regimes

Though other patterns exist, generally four horizontal regimes are defined: Bubble, Slug, Annular and Mist flow

Plug/Bubble Flow

- Pipe is almost completely filled with liquid and the free gas phase is present in small bubbles.
- Pipe wall is always contacted by the liquid phase

Slug or Intermittent Flow

- Liquid phase with accompanying gas bubble gas

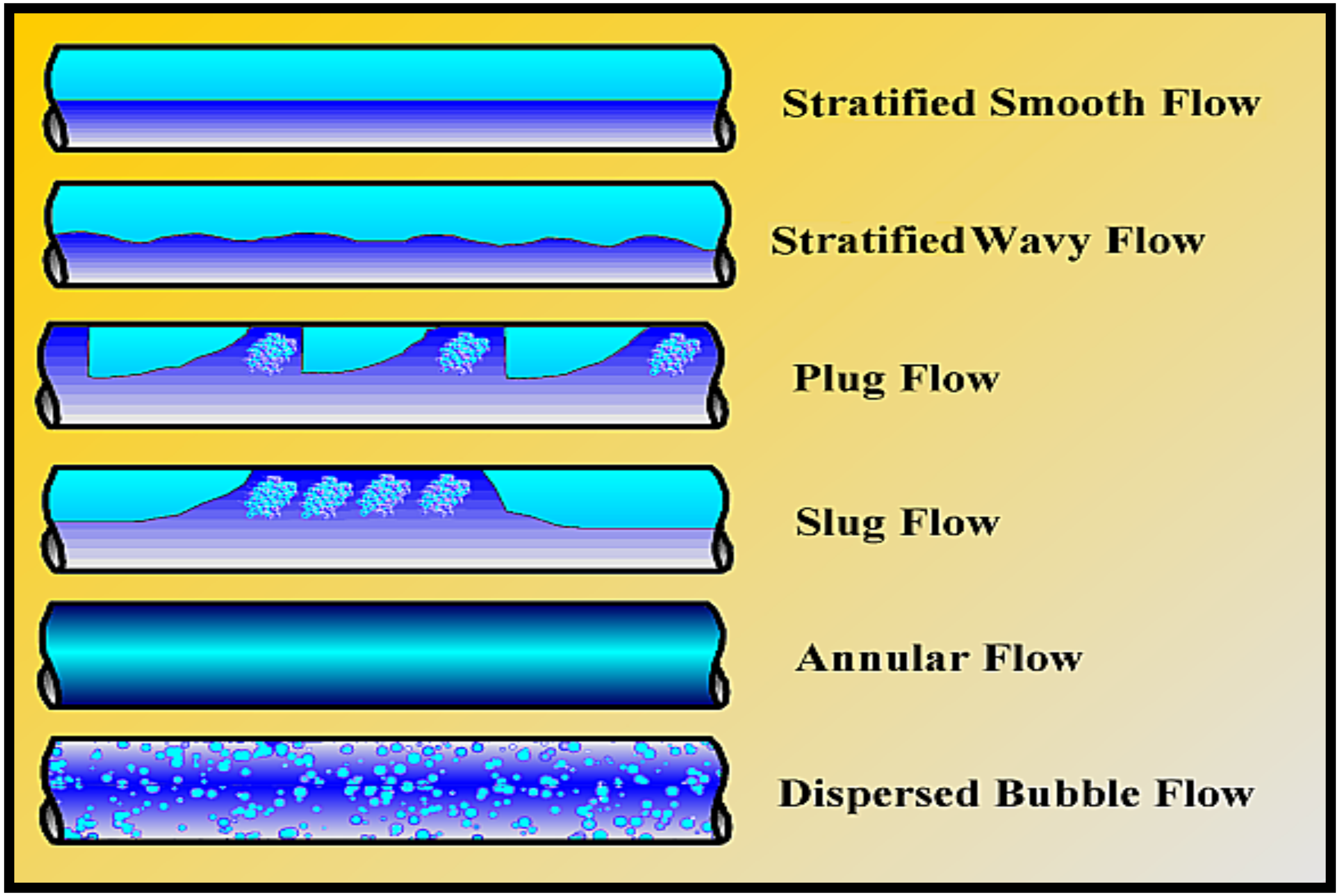
Stratified Flow

- Pipe acts as a separator
- Liquid flows along bottom with gas above

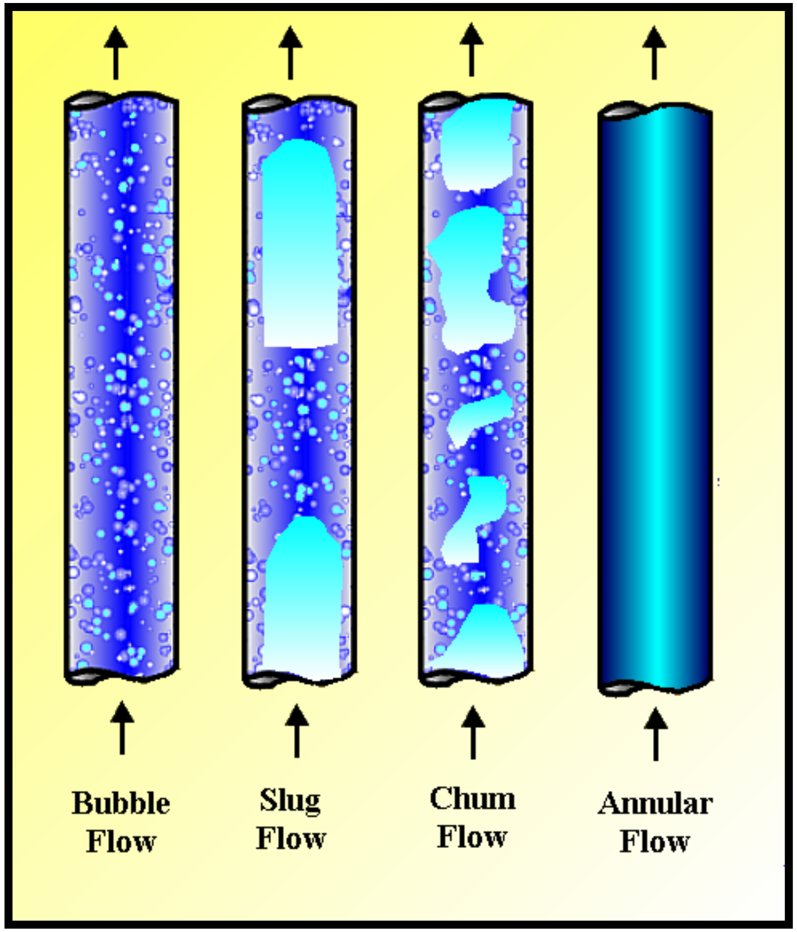
Annular Flow

- High velocity gas core
- Liquid annulus surrounds a gas core

Horizontal Flow Regimes



Vertical Flow Regimes



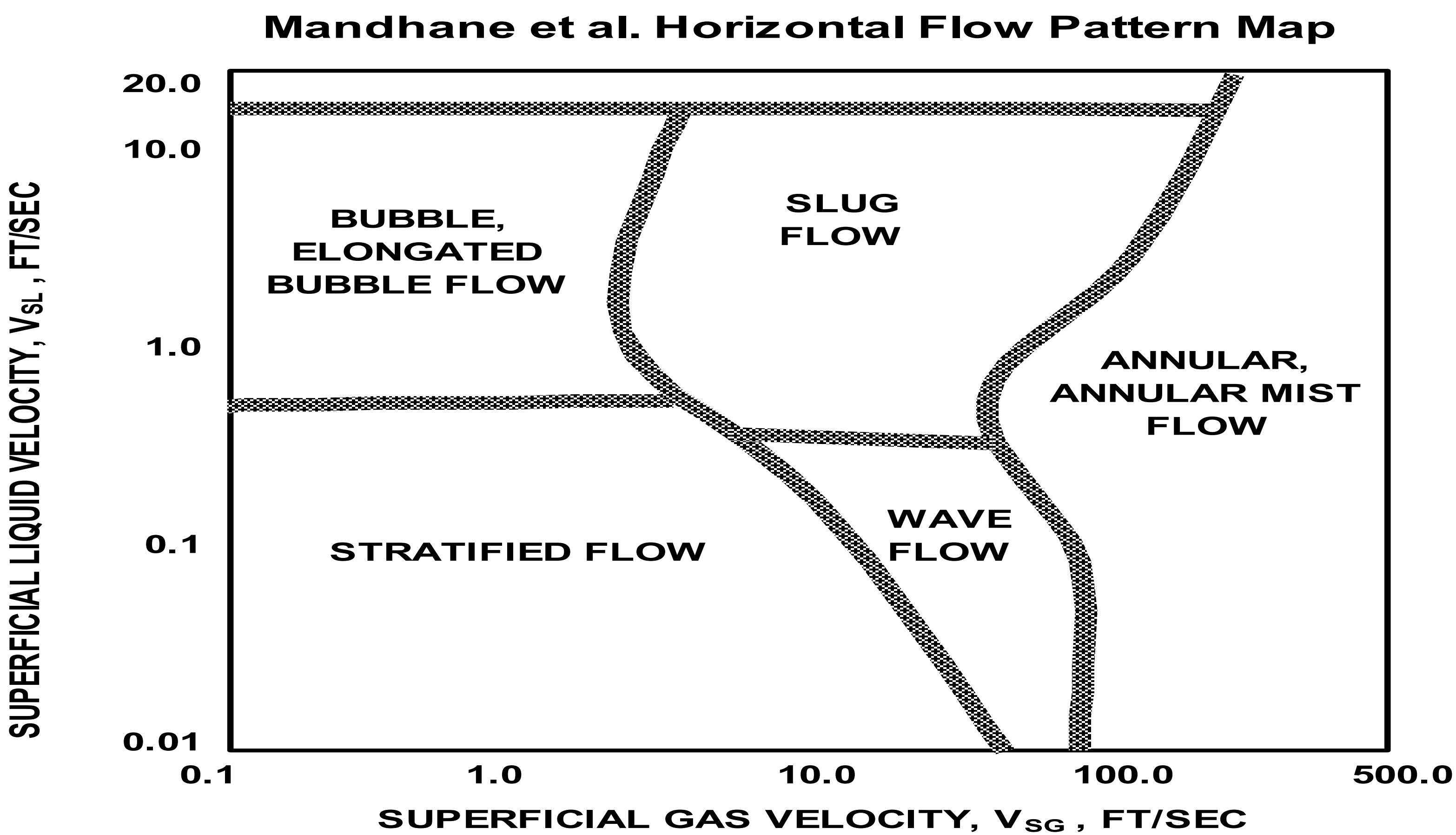
Flow Regimes

Experimental studies of flow regime transitions have shown that each of the flow regime boundaries reacts differently to changes in the system variables. The following table shows the sensitivity of the transitions to changes in the major system variables:

Transition Variable	Slug to Dispersed Bubble	Slug to Annular	Slug to Stratified	Stratified to Annular
Angle of Inclination	Small Effect	Moderate Effect	Strong Effect	Strong Effect
Gas Density	Small Effect	Strong Effect	Strong Effect	Strong Effect
Pipeline Diameter	Small Effect	Small Effect	Strong Effect	Moderate Effect
Liquid Physical Properties	Moderate Effect	Small Effect	Moderate Effect	Moderate Effect

Many researchers have attempted to develop simple flow regime maps, usually using dimensionless parameters on each axis (e.g. Baker, Beggs & Brill). These methods can be inaccurate since no single parameter can model the sensitivity effects shown in the previous table.

Mandhane Flow Regime Map



Early flow map developed mainly from air water systems. Scale up to oilfield conditions uncertain.

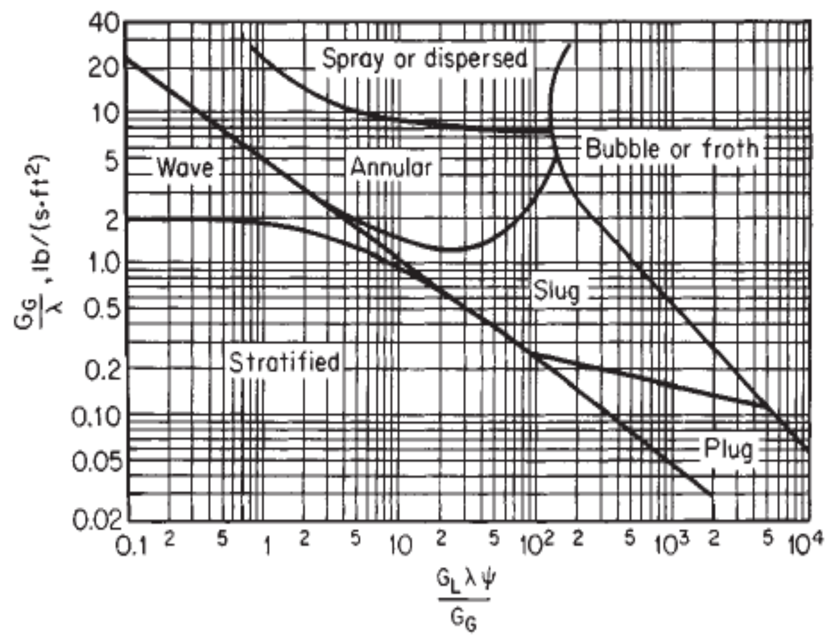
Baker Flow Regime Map

The Baker Flow Regime map incorporates physical properties not included within the Mandhane chart. In the Baker chart, G_L and G_G are the liquid and gas mass velocities, μ^{l}_{L} is the ratio of liquid viscosity to water viscosity, μ^{l}_{G} is the ratio of gas density to air density, ρ^{l}_{L} is the ratio of liquid density to water density, and σ^{l}_{L} is the ratio of liquid surface tension to water surface tension.

The reference properties are at 20°C (68°F) and atmospheric pressure, water density 1,000 kg/m3 (62.4 lbm/ft3), air density 1.20 kg/m3 (0.075 lbm/ft3), water viscosity 0.001 Pa × s, (1.0 cp) and surface tension 0.073 N/m (0.0050 lbf/ft, 73 dyne/cm). The empirical parameters λ and ψ provide a crude accounting for physical properties.

$$\lambda = (\rho'_G \rho'_L)^{1/2}$$

$$\psi = \frac{1}{\sigma'}\left[\frac{\mu'_L}{(\rho'_L)^2}\right]^{1/3}$$



Flow-pattern regions in cocurrent liquid/gas flow through horizontal pipes. To convert lbm/(ft² · s) to kg/(m² · s), multiply by 4.8824. (*From Baker, Oil Gas J.*, **53**[*12*], *185–190, 192, 195* [*1954*].)

Worked Example – Mandhane and Baker Map

Example – Flow Regime

Pipeline is flowing with the following conditions

$d_i = 15 \text{ ins} = 0.381 \text{ m}$

$Q_g = 29.5 \text{ ft}^3/s = 0.835 \text{ m}^3/s$

$Q_l = 1.3 \text{ ft}^3/s = 0.0368 \text{ m}^3/s$

$\rho_g = 8.39 \text{ lb/ft}^3 = 134.4 \text{ kg/m}^3$

$\rho_l = 35.57 \text{ lb/ft}^3 = 569.8 \text{ kg/m}^3$

$\mu_g = 0.017 \text{ cp} = 0.000017 \text{ Pa.S.}$

$\mu_l = 0.833 \text{ cp} = 0.000833 \text{ Pa.s.}$

$\sigma = 5 \text{ dyne/cm} = 0.005 \text{ N/m.}$

Calculate pipe C.S.A. $= 0.1139 \text{ m}^2$

∴ Superficial gas velocity, $v_{sg} = 7.33 \text{ m/s}$

∴ liquid " , $v_{sl} = 0.323 \text{ m/s}$

∴ Mixture velocity, $v_m = v_{sg} + v_{sl} = 7.65 \text{ m/s}$

Reference properties

$\rho_L = 1000 \text{ kg/m}^3$

$\rho_g = 1.2 \text{ kg/m}^3$

$\mu_L = 0.001 \text{ Pa.s}$

$\sigma = 0.073 \text{ N/m.}$

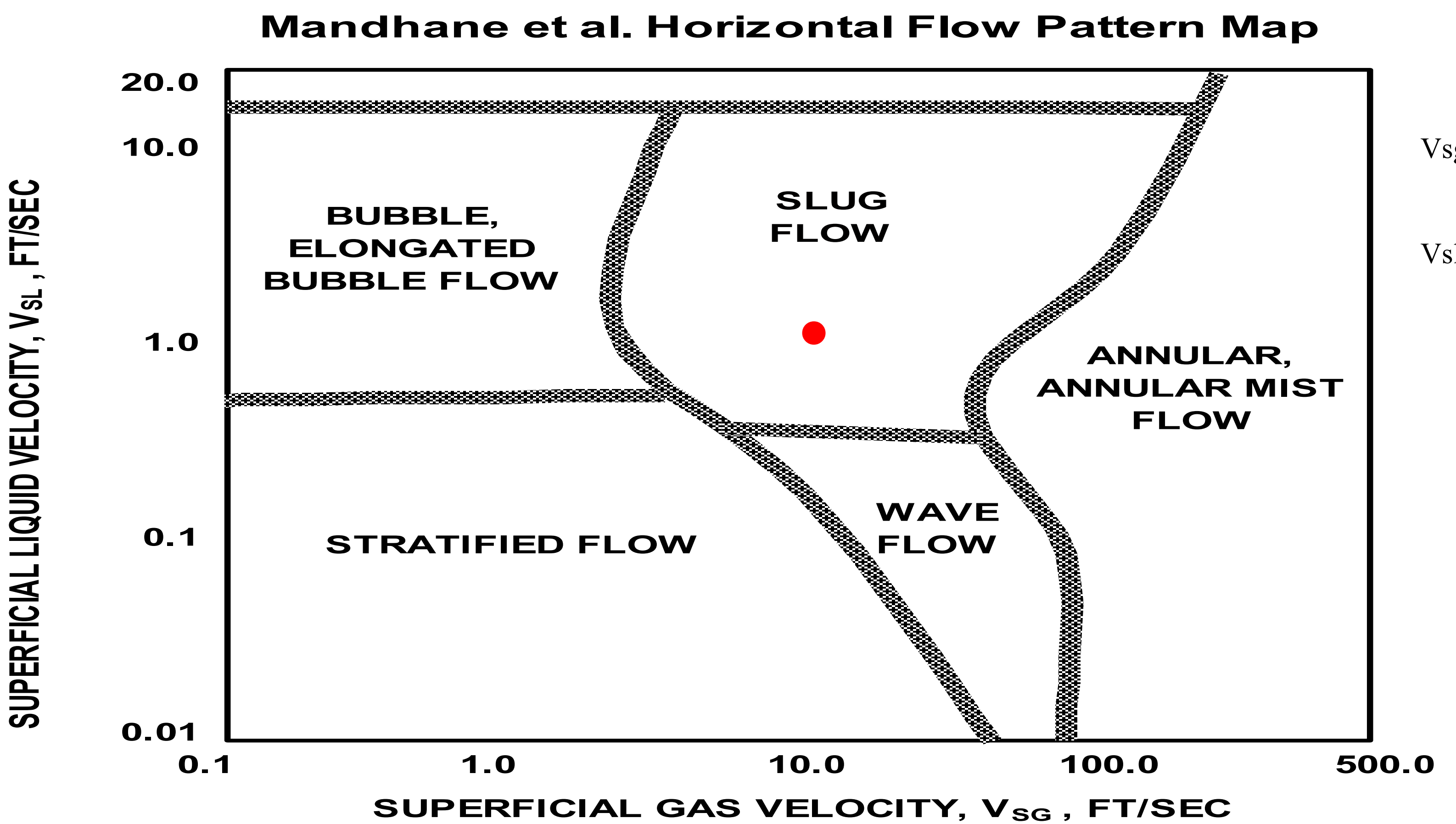
$\rho_g' = \frac{134.4}{1.2} = 112$

$\rho_L' = \frac{569.8}{1000} = 0.57$

$\mu_L' = \frac{0.000833}{0.001} = 0.833$

$\sigma' = \frac{0.005}{0.073} = 0.068$

Worked Example - Mandhane Flow Regime Map



V_{sg} = 7.33 m/s
= 24 ft/s

V_{sl} = 0.323 m/s
= 1.06 ft/s

Only require superficial velocities - Flow pattern is slug.

Worked Example – Baker Map

$$\lambda = \left(\rho_g' \cdot \rho_L'\right)^{1/2} = 8.0$$

$$\psi = \frac{1}{\sigma'}\left(\frac{\mu_L'}{(\rho_L')^2}\right)^{1/3} = 20.0$$

Gas mass velocity, $G_g = \frac{Q_g \cdot \rho_g}{C.S.A} = 985.1\ kg/m^2 s$ $\left[\frac{\cancel{m^3}}{s} \cdot \frac{kg}{\cancel{m^3}} \cdot \frac{1}{m^2}\right]$

Liquid mass velocity, $G_L = \frac{Q_L \cdot \rho_L}{CSA} = 184.0\ kg/m^2 s$

$$\frac{G_g}{\lambda} = \frac{985.1}{8} = 123.1\ kg/m^2 s = 25.2\ lb/ft^2 s$$

$$\frac{G_L}{G_G} \cdot \lambda \cdot \psi = \frac{184.0}{985.1} \times 8.0 \times 20.0 = 29.8$$

Worked Example – Baker Map

$$\lambda = (\rho'_G \rho'_L)^{1/2} \qquad (6\text{-}129)$$

$$\psi = \frac{1}{\sigma'}\left[\frac{\mu'_L}{(\rho'_L)^2}\right]^{1/3} \qquad (6\text{-}130)$$

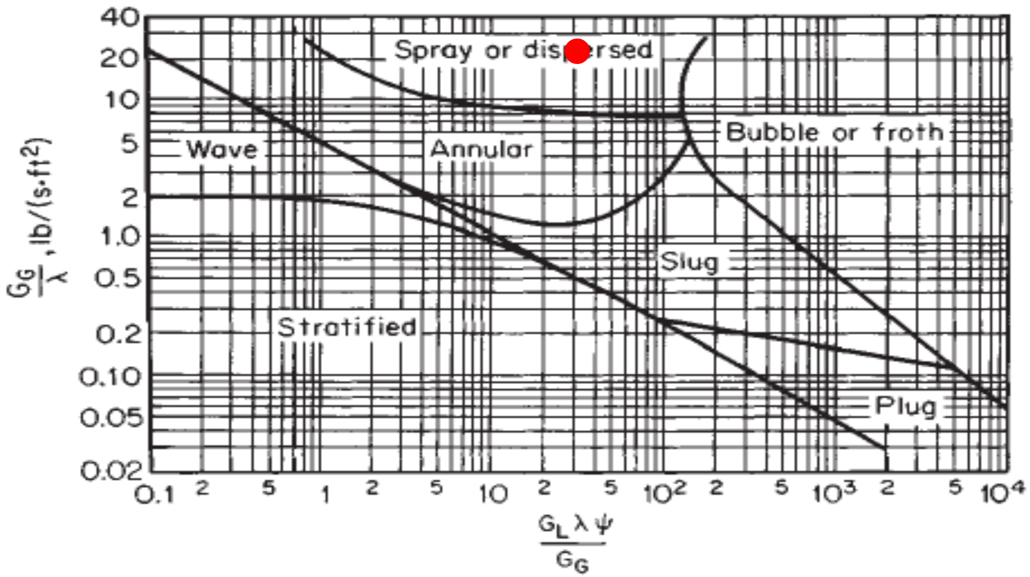


FIG. 6-25 Flow-pattern regions in cocurrent liquid/gas flow through horizontal pipes. To convert lbm/(ft² · s) to kg/(m² · s), multiply by 4.8824. (*From Baker, Oil Gas J.,* **53**[12], *185–190, 192, 195* [1954].)

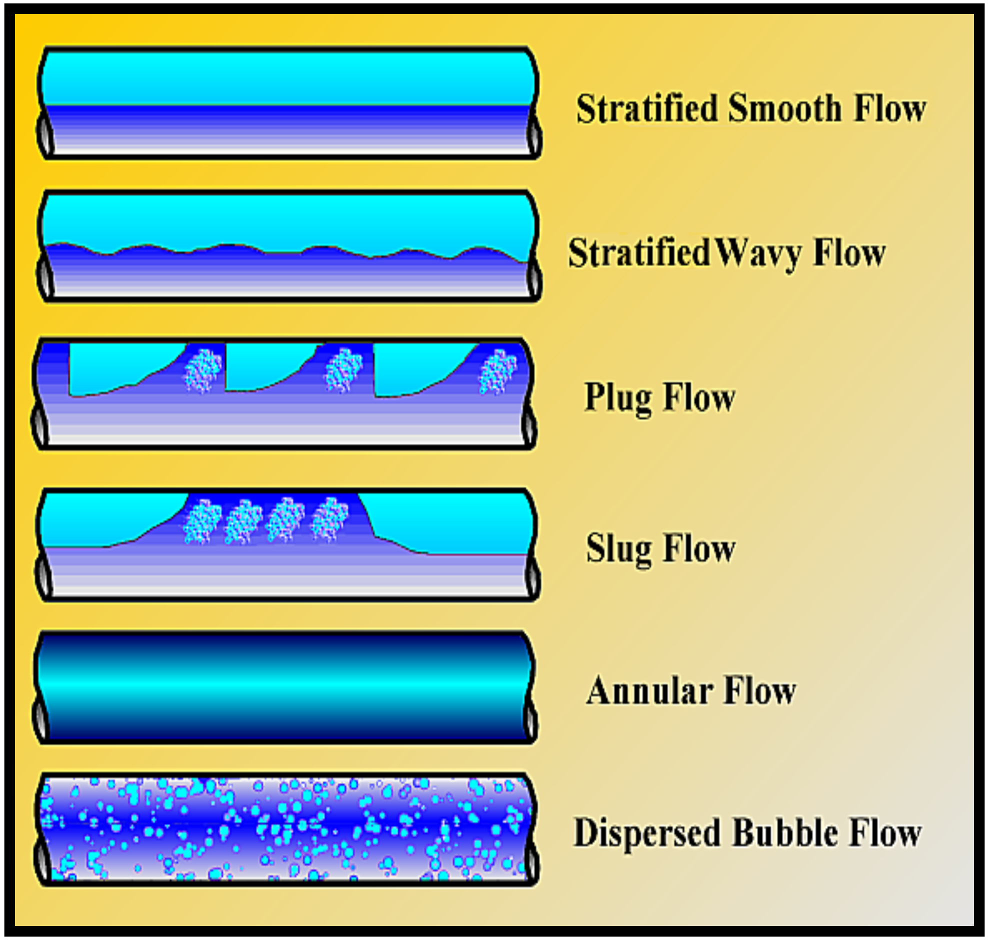
Flow pattern is dispersed bubble.

Flow Regime Map

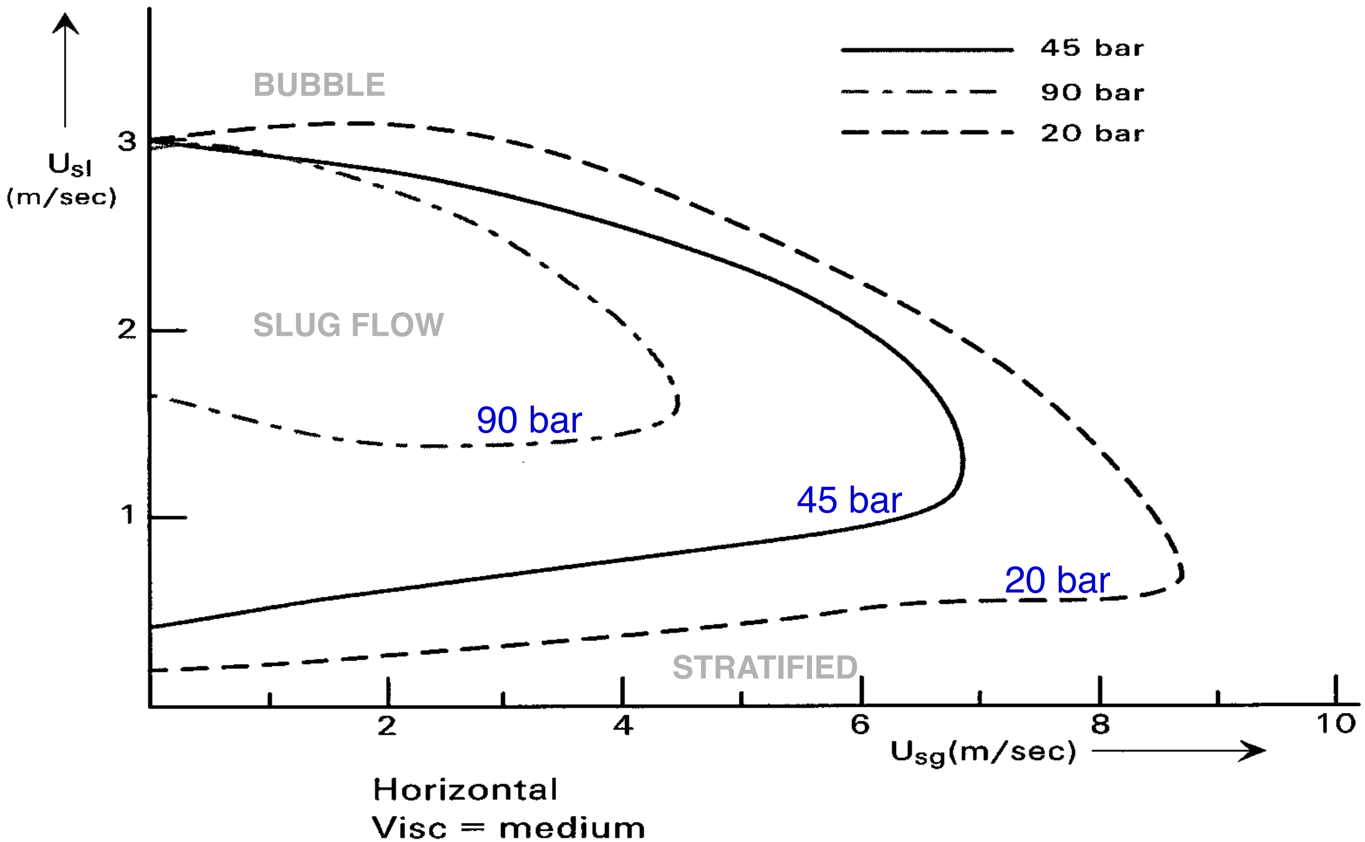
Mandhane – Slug

Baker - Dispersed Bubble

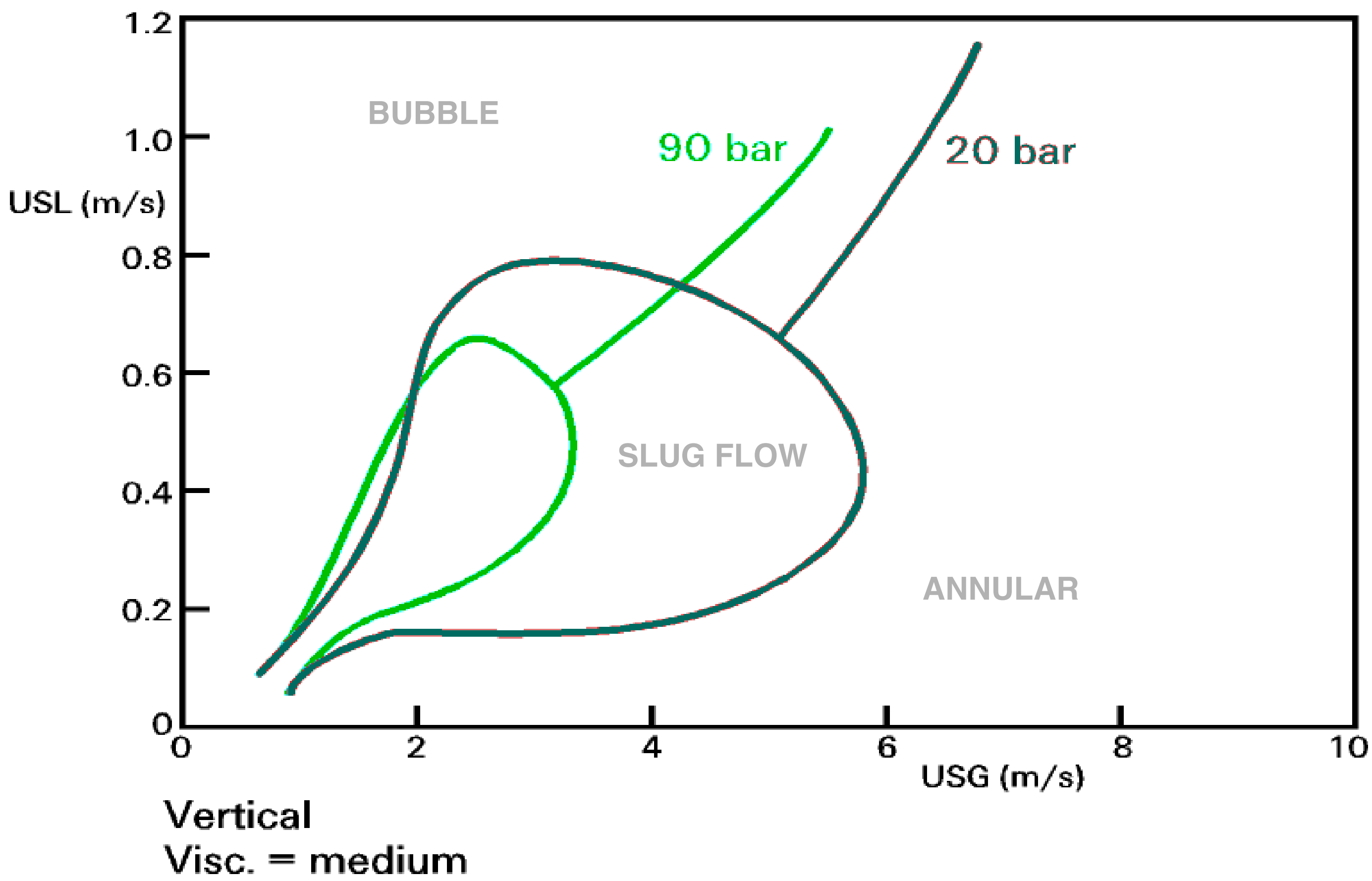
What is correct?



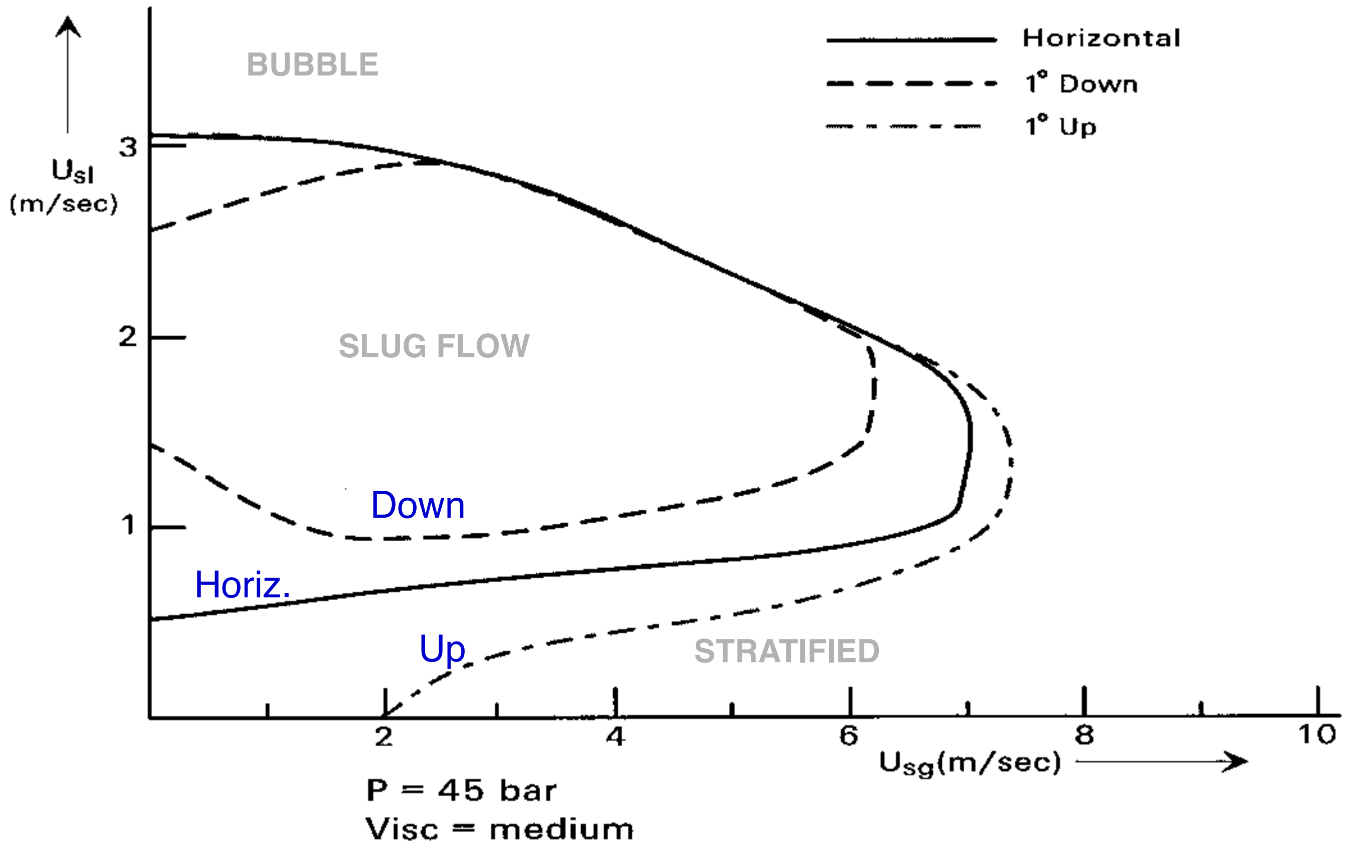
Pressure impact on flow regime
(Horizontal flow)



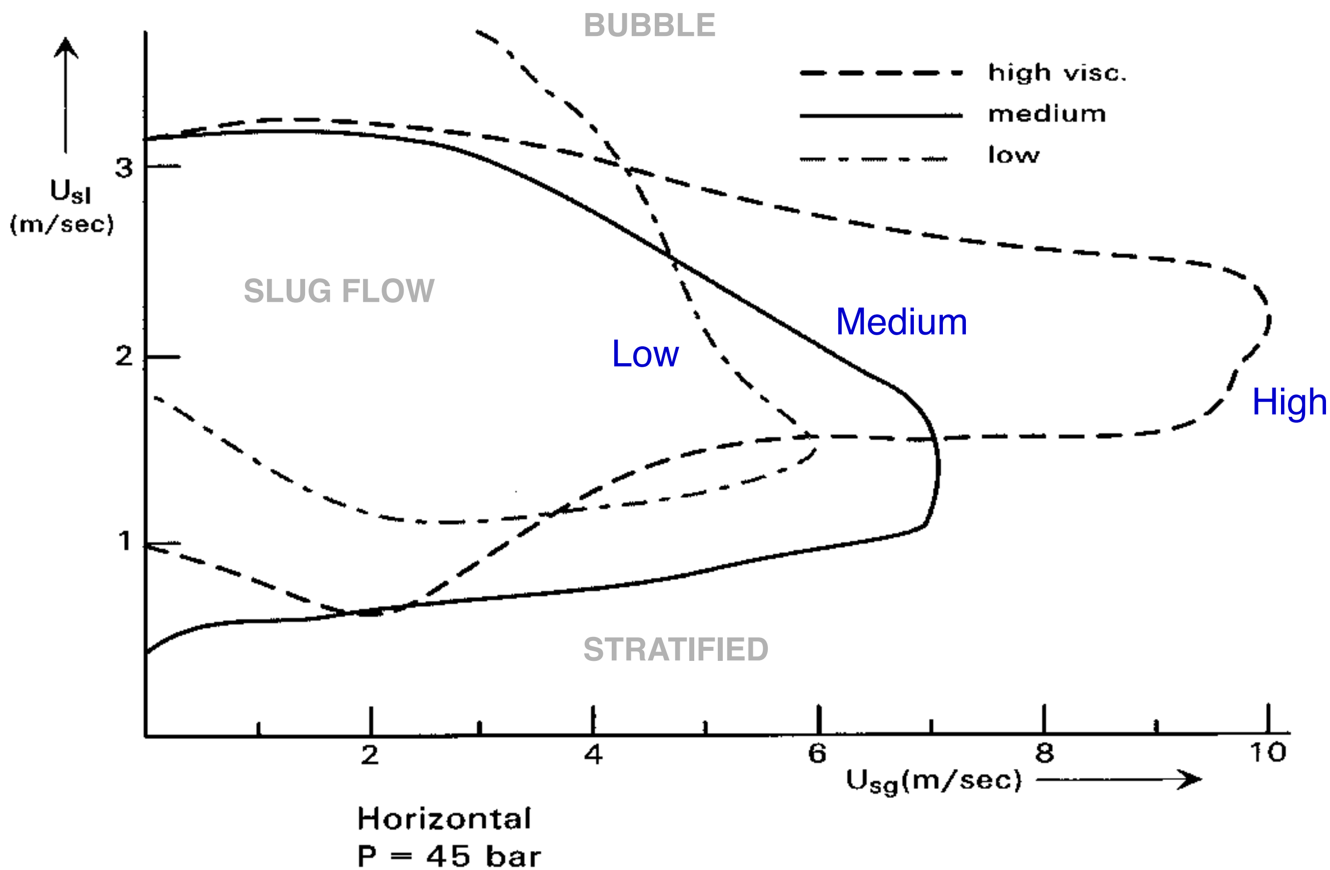
Pressure impact on flow regime
(Vertical flow)



Inclination impact on flow regime



Viscosity impact on flow regime



Significant impact on flow regime boundaries – from physical properties and pipe angle – early models inaccurate for oilfield applications.

Froude Number

The Froude number (F or Fr), is a dimensionless quantity used to indicate the influence of gravity on fluid motion. It is generally expressed as $Fr = v/(gd)^{0.5}$, in which d is depth of flow or diameter, g is the gravitational acceleration (equal to the specific weight of the water divided by its density, in fluid mechanics), v is the velocity of a small surface (or gravity) wave, and Fr is the Froude number. When Fr is less than 1, small surface waves can move upstream; when Fr is greater than 1, they will be carried downstream; and when Fr = 1 (said to be the critical Froude number), the velocity of flow is just equal to the velocity of surface waves.

Taitel –Dukler Flow Regime

Taitel and Dukler made the first attempt at understanding the underlying the physics of flow transitions. The map uses the Martinelli parameter X, the gas Froude number, Fr and the parameters T and K. The map is composed of three graphs as shown later.

The Martinelli parameter is

$$X^2 = \frac{(dp/dz)_{Fsf}}{(dp/dz)_{Fsg}}$$

Pressure gradient if liquid alone flowing

Pressure gradient if gas alone flowing

and the gas-phase Froude number is

$$F = \sqrt{\frac{\rho_g}{\rho_f - \rho_g}} \frac{j_g}{\sqrt{Dg\cos\theta}}$$

The parameter T is defined as

$$T = \left[\frac{\left|(dp/dz)_{sf}\right|}{(\rho_f - \rho_g) g \cos\theta} \right]^{1/2}$$

j_g = is the gas superficial velocity = $G .x / \rho_g$
G = mass velocity kg/m^{2}s
x = mass vapour quality – vapour mass fraction
θ = pipe angle

Taitel –Dukler Flow Regime

and the parameter K is

$$K = F\left(\frac{D\, j_f}{\nu_f}\right)^{1/2} = F\, \mathrm{Re}_{sf}^{\ 1/2}$$

Kinematic viscosity

where the liquid-phase and gas-phase Reynolds number are

$$\mathrm{Re}_f = \frac{G_f D}{\mu_f}$$

G is mass flux.

and

$$\mathrm{Re}_g = \frac{G_g D}{\mu_g}$$

The pressure gradient of the flow for phase k (where k is either f or g) i

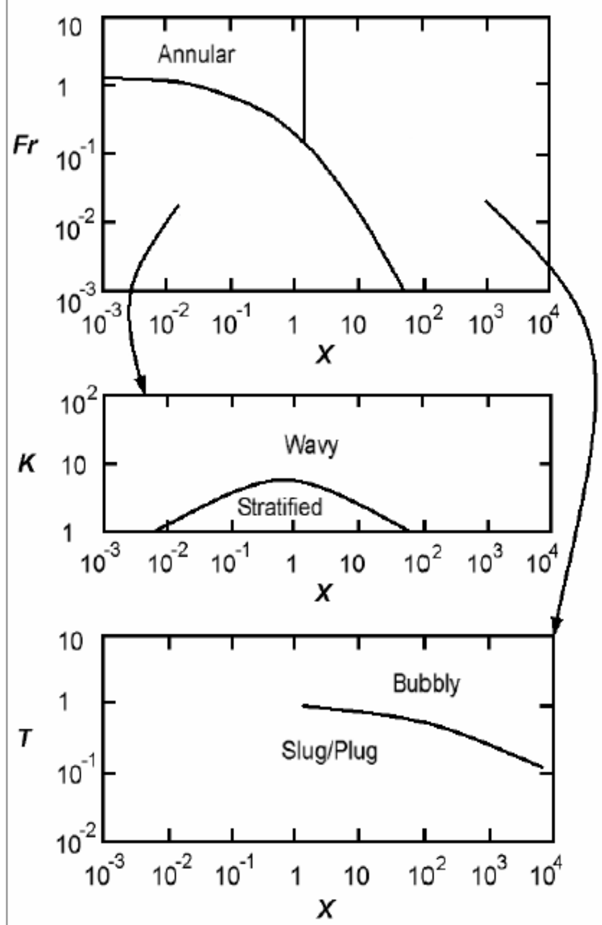
$$(dp/dz)_k = -\frac{2 f_k G_k^{\ 2}}{\rho_k D}$$

Taitel –Dukler Flow Regime

A recommended procedure of pattern determination using the Taitel and Dukler (1976) map is:

- First determine the Martinelli parameter X and F
- Using the two parameters on the top graph, if the coordinates fall in the annular flow regime, then the flow pattern is *annular*. Otherwise
- If the coordinates of X and Fr fall in the lower left zone of the top map, then calculate K.
  - Using K and X in the middle graph, the flow regime is identified as either *stratified-wavy* or as *fully stratified*.
- If the coordinates Fr and X fall in the right zone on the top graph, then T is calculated.
  - Using T and X in the bottom graph, the flow regime is identified as either *bubbly flow* or *intermittent (plug or slug)* flow.

A worked example is included at the end of the lecture notes.

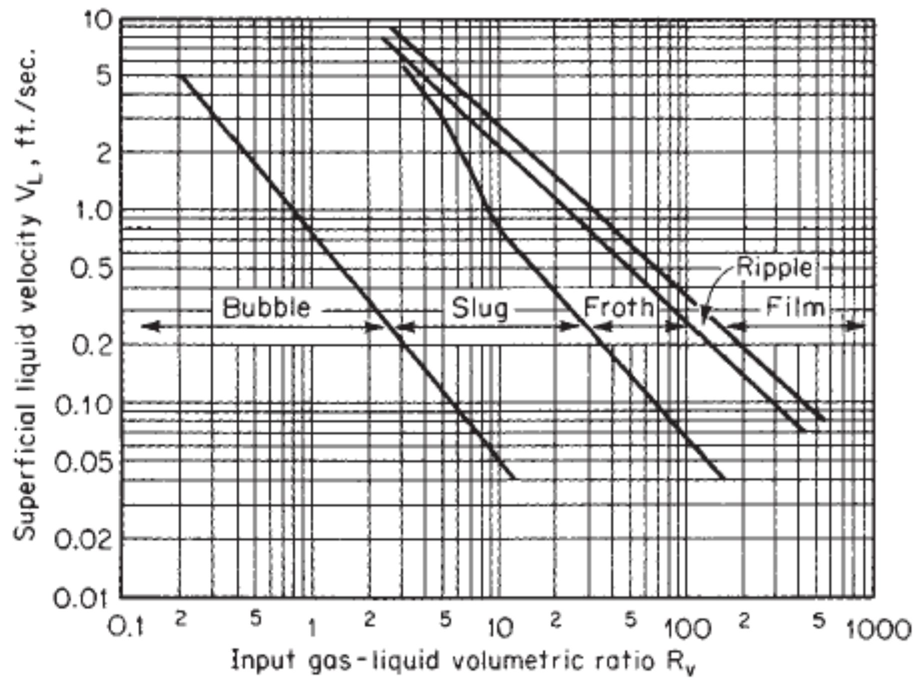


Govier Vertical Multi-Phase Map

The correlation by Govier, et al. (Can. J. Chem. Eng., 35, 58–70 [1957]), may be used for quick estimate of flow pattern.

$R_v = V_{sg}/V_{sl}$

Film flow is annular flow
Froth flow is churn flow



Flow-pattern regions in cocurrent liquid/gas flow in upflow through vertical pipes. To convert ft/s to m/s, multiply by 0.3048. (*From Govier, Radford, and Dunn,* Can. J. Chem. Eng., **35,** *58–70* [*1957*].)

Worked Example – Vertical Flow

Example – Flow Regime

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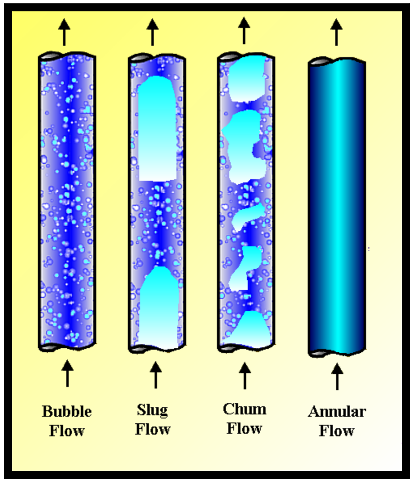
$\mu_L' = \frac{0.000833}{0.001} = 0.833$

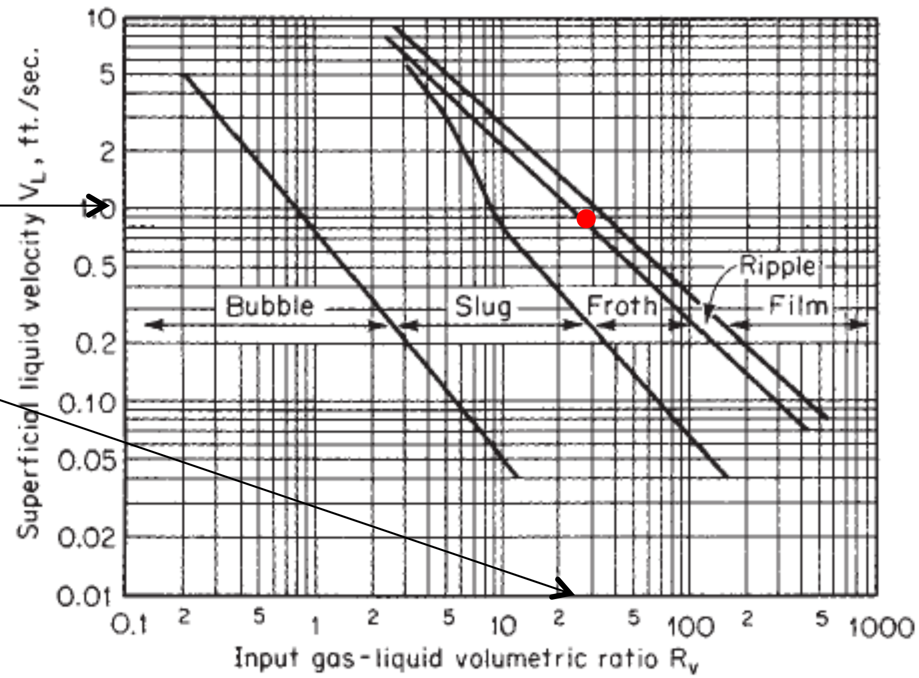
$\sigma' = \frac{0.005}{0.073} = 0.068$

Worked Example – Vertical Flow

Vsg = 7.33 m/s
  = 24 ft/s
Vsl = 0.323 m/s
  = 1.06 ft/s
Vsg/ Vsl = 24

Ripple/film boundary



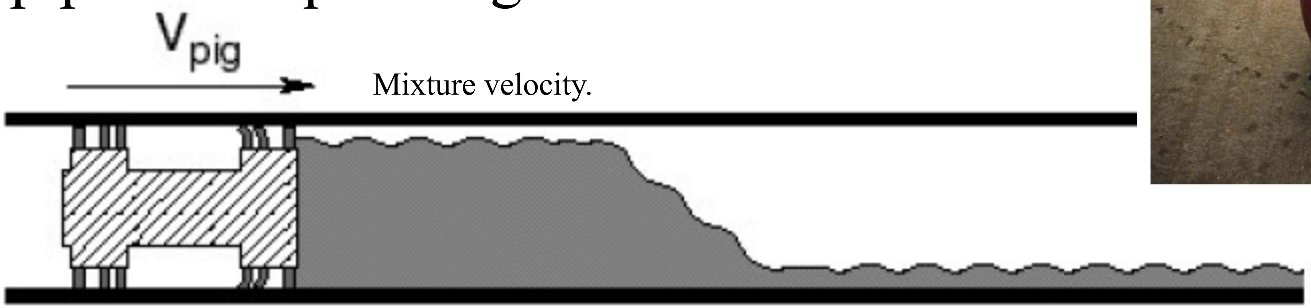


Flow-pattern regions in cocurrent liquid/gas flow in upflow through vertical pipes. To convert ft/s to m/s, multiply by 0.3048. (*From Govier, Radford, and Dunn*, Can. J. Chem. Eng., **35**, *58–70* [*1957*].)

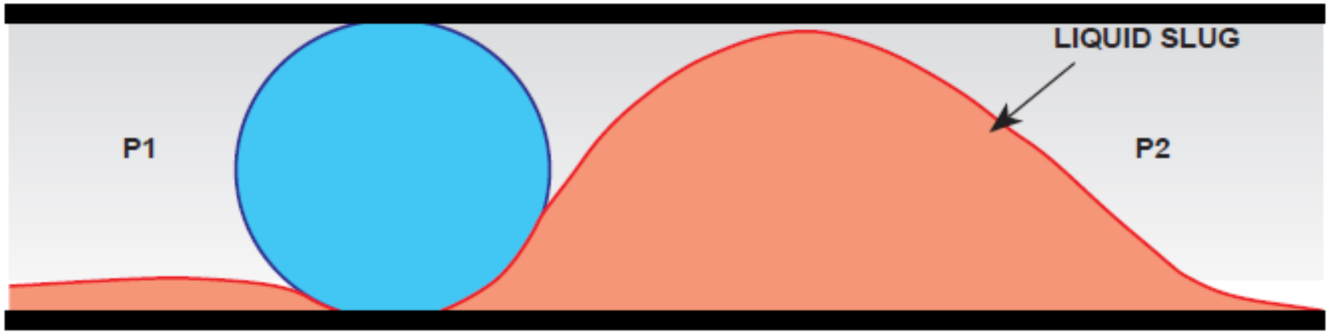
Wet Gas Pipeline Pigging and Slug Catcher

Pipeline Sphering

Improve pipeline capacity by removing liquid. The liquid is displaced at the end of the pipe as a liquid slug.

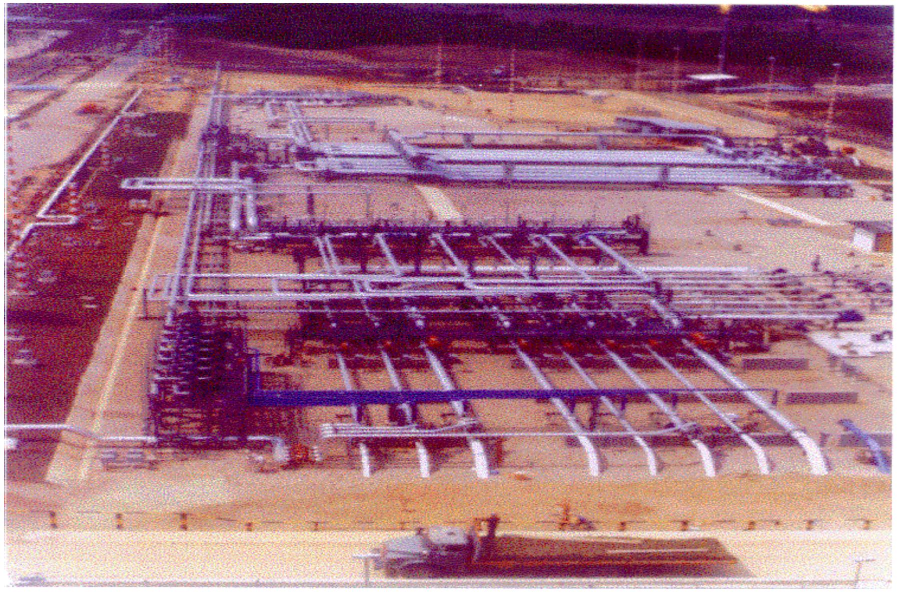


Sphere

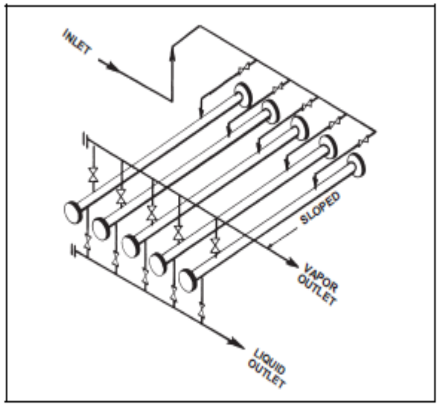


P1 IS GREATER THAN P2

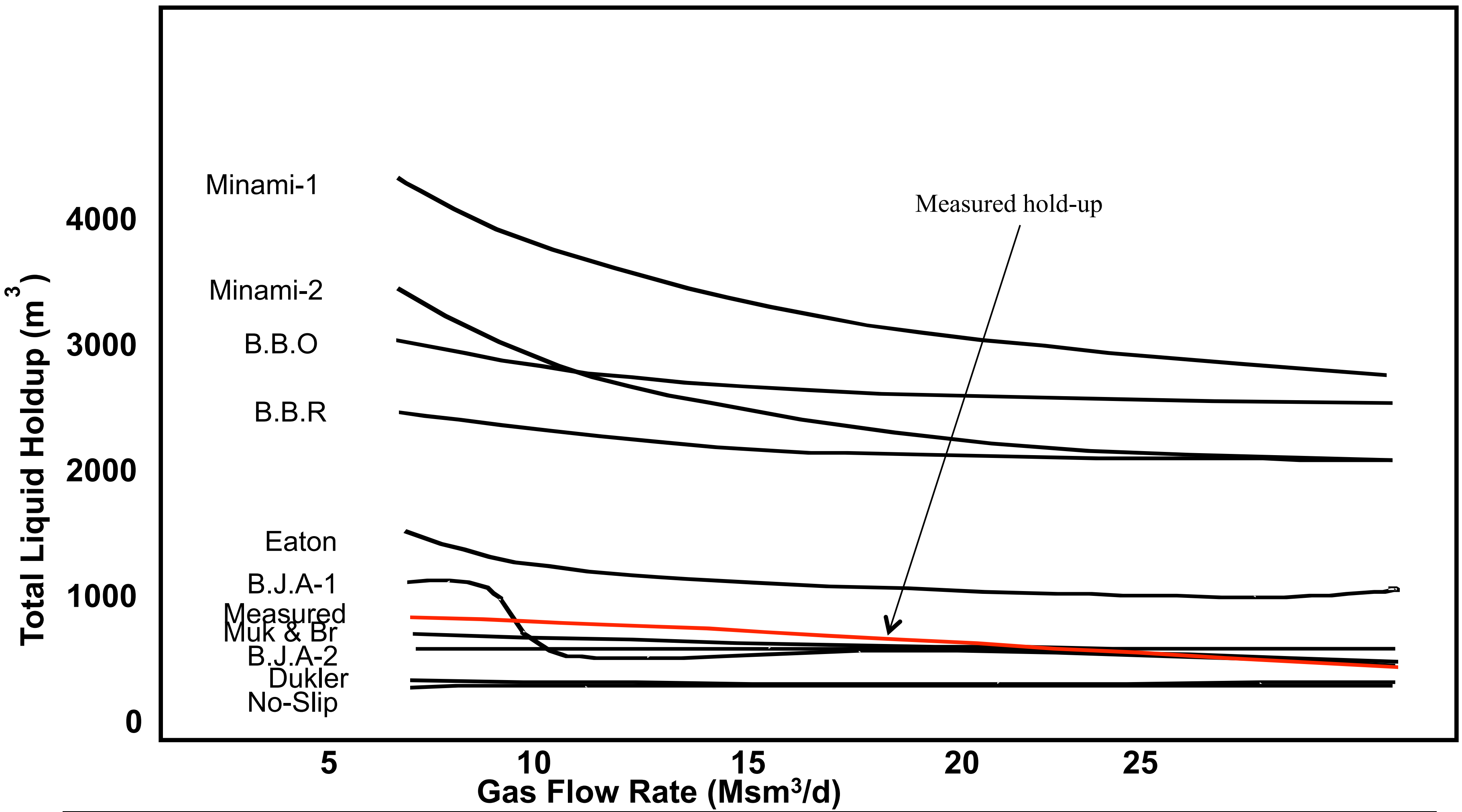
Wet Gas Pipeline Slug Catcher



Multiple Pipe Slug Catcher



Hold Up – Wide Variation in predicted values



Hold up – Eaton Method

A correlation developed by Eaton is widely used for estimating hold-up in pipe lines flowing stratified. The Eaton holdup correlation is shown in the opposite figure. In this figure, the holdup fraction is plotted directly as a function of a dimensionless group. The dimensionless group are of the form:

$$N_e = \frac{1.84\,(N_{Lv})^{0.575}\left(\frac{P_{avg}}{P_b}\right)^{0.05}(N_L)^{0.1}}{N_{gv}\,(N_d)^{0.0277}}$$

where

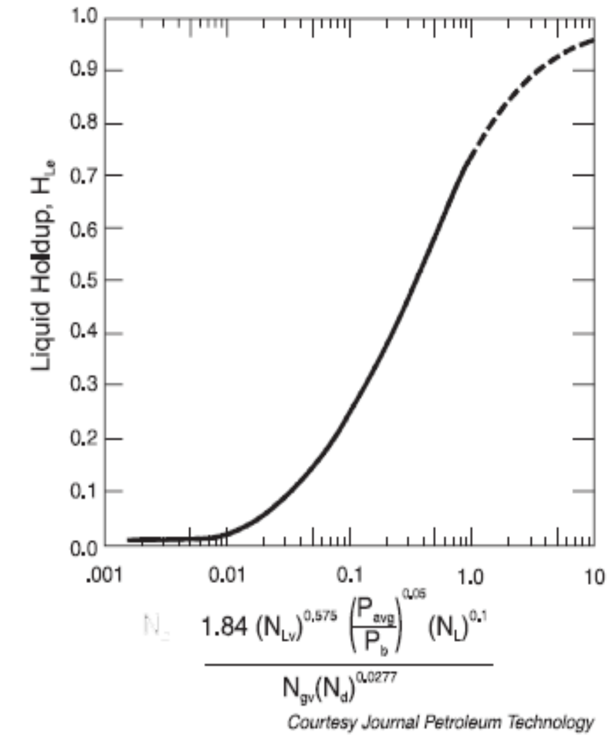
$$N_{Lv} = 0.0565\,V_{sL}\left(\frac{\rho_L}{\sigma}\right)^{0.25}$$

$$N_{gv} = 0.0565 V_{sg}\left(\frac{\rho_L}{\sigma}\right)^{0.25}$$

$$N_d = 0.00003134\,d\left(\frac{\rho_L}{\sigma}\right)^{0.50}$$

$$N_L = 0.001769\,\mu_L\left(\frac{1}{\rho_L\sigma^3}\right)^{0.25}$$

Eaton Liquid Holdup Correlation



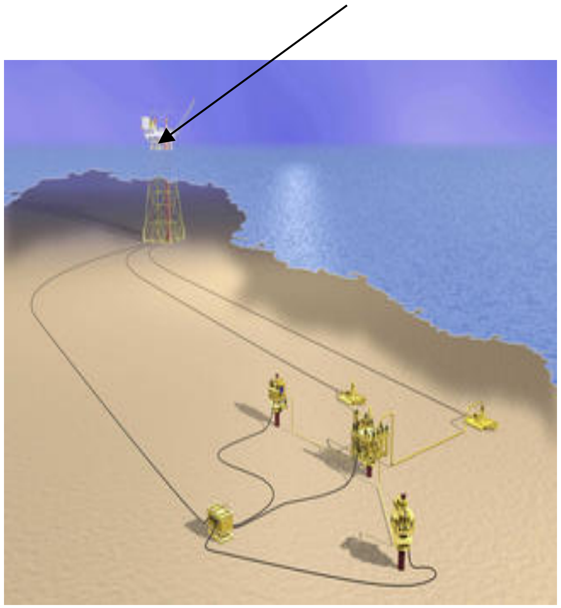
A worked example is provided at the end of the lecture notes. Note units of d are mm.

Multi-Phase Flow Pressure Drop Correlation Development

Accurate prediction of pressure drop in a two/multi-phase system is an essential element of flow assurance. Inaccurate analysis of pressure loss will mean that the pipeline is over or undersized which on both counts could lead to loss of production and loss of revenue together with serious operational problems.

The boundary pressure is often an existing pressure on a host installation. The sub sea architecture must be designed to ensure that the production profile can be delivered to the host pressure inlet requirements.

Using the host pressure as a boundary, calculations are undertaken to quantify required flowing wellhead pressures. These are compared with flowing wellhead pressures established from well lift curves to ensure there is adequate wellhead pressure to deliver the production fluids.



Multi-Phase Flow Pressure Drop Correlation Development

There are three categories of flow correlations

Category 1

- No-slip, no flow regime considered
- Mixture density and viscosity based on input average phase densities
- Gas and liquid assumed to be travelling at same velocity in pipe
- Only require a two-phase friction factor correlation
- No distinction made for different flow regimes.

Example:

- Lockhart Martinelli
- Poettmann and Carpenter
- Baxendell and Thomas
- Fancher and Brown
- Homogeneous Flow Model

Category 2

- Slip considered, no flow regime considered
- Gas and liquid travel at different velocity in pipe
- Requires a two-phase friction factor correlation and liquid holdup correlation
- No distinction made for different flow regimes.

Example:

- Hagedorn and Brown

Multi-Phase Flow Pressure Drop Correlation Development

Category 3

- Slip considered, Flow Regime considered
- Requires method to predict flow regime
- Requires specific two-phase friction factor and liquid holdup correlations for each flow regime
- Acceleration pressure gradient depends on flow regime

Example:

- Duns and Ros
- Orkiszewski
- Aziz, Govier and Fogarasi
- Beggs and Brill

The correlations reflect the complexities of multi-phase flow with all work being very empirical in nature.

Multi-Phase Flow
Category 1 – Homogeneous flow model

The simplest approach to the prediction of two-phase flows is to assume that the phases are thoroughly mixed and can be treated as a single-phase flow. This homogeneous model will obviously work best when the phases are strongly inter-dispersed (i.e. at high velocities). This type of model utilises a two-phase friction factor calculated from a two-phase Reynold's number.

In determining two-phase Reynolds number the following equations is often used to calculate a two-phase viscosity.

$$\mu_n = \mu_l . \lambda_l + \mu_g . \lambda_g$$

Similarly a no slip two-phase density is utilised.

$$\rho_n = \rho_l . \lambda_l + \rho_g . \lambda_g$$

Recall λ_l and λ_g are the no slip liquid and gas hold up.

Two-Phase Flow
Category 1 – Homogeneous flow model

The two-phase Reynold's number becomes;

$Re_n = \rho_n . v_m . d / \mu_n$

Where v_m is the no slip mixture velocity.

The two – phase friction factor can be determined using the Blasius expression;

$f_{tp} = 0.079 . Re_n^{-0.25}$

Two-frictional pressure drop becomes;

$\Delta P_f = 2 . f_{tp} . \rho_n . v_m^2 / d$

Homogeneous flow model
Example calculation

Calculate the two phase pressure drop for vertical flow in a pipe, 10 mm internal diameter, 20 m long.
The mass flow rate is 0.20 kg/s and vapour mass quality is 0.05. The fluid properties are as follows:

$$\rho_L = 1518 \ kg/m^3$$

$$\rho_V = 2.60 \ kg/m^3$$

$$\mu_L = 0.0005856 \ kg/m.s$$

$$\mu_g = 0.0000126 \ kg/m\,s$$

Homogeneous flow model

2.

Calculate superficial velocities ;

Vol. flow liquid, $V_L = \frac{0.20 \times (1 - 0.05)}{1518} = 1.25 \times 10^{-4}\ \frac{kg}{s} \times \frac{m^3}{kg} = m^3/s$.

Vol. flow gas, $V_g = \frac{0.20 \times 0.05}{2.60} = 3.85 \times 10^{-3}\ m^3/s$

Pipe CSA, $A = \frac{\pi D^2}{4} = \frac{\pi \times 0.01^2}{4} = 7.85 \times 10^{-5}\ m^2$

$\therefore$ superficial liquid velocity, $v_{sL} = \frac{1.25 \times 10^{-4}}{7.85 \times 10^{-5}} = 1.59\ m/s$

" gas " , $v_{sg} = 49.0\ m/s$.

$\therefore$ no slip hold up $= \frac{1.59}{1.59 + 49.0} = 0.0314$

Homogeneous flow model

3

No slip density, $\rho_n = 0.0314 \times 1518 + (1-0.0314) \times 2.6$

$= 47.7 + 2.52$

$= 50.2 \ kg/m^3$

Two-phase viscosity, $\mu_n = 0.0314 \times 0.000586 + (1-0.0314) \times 0.0000216$

$= 0.0000184 + 0.00002092$

$= 0.0000393 \ kg/ms$

Mixture velocity, $v_m = v_{sc} + v_{sc}$

$= 1.59 + 49.0$

$= 50.59 \ m/s$.

∴ Two-phase Reynolds number, $Re_n = \dfrac{\rho_n v_m \cdot d}{\mu_m} = \dfrac{50.2 \times 50.59 \times 0.01}{3.93 \times 10^{-5}}$

$= 6.46 \times 10^5$

Homogeneous flow model

4.

Two-phase friction factor, $f_{tp} = \frac{0.079}{Re_n^{0.25}} = \frac{0.079}{(6.46 \times 10^5)^{0.25}}$

$= 0.00278.$

$\therefore \Delta P_f = \frac{2 . f_{tp} . \rho_n . v_m^2}{d}$

$= \frac{2 \times 0.00278 \times 50.2 \times 50.59^2}{0.01}$

$= 71,400 \ N/m^2 \ /m$

$= 71.4 \ kPa/m^2 \ /m.$

$L = 20 \ m.$

$\therefore \Delta P_f = 20 \times 71.4$

$= 142.8 \ kPa \quad (1.428 \ bar)$

Homogeneous flow model

Elevation pressure loss. 5

$$\Delta P_E = \rho_h \cdot g \cdot h$$

$$= 50.2 \times 9.81 \times 20$$

$$= 9850 \ N/m^2 \ (Pa)$$

$$= 9.85 \ kPa$$

Assuming acceleration losses are negligible

$$\Delta P_{TOT} = \Delta P_f + \Delta P_E$$

$$= \underline{\underline{152.6 \ kPa}}$$

Multi-Phase Flow
Category 1 – Lockhart and Martinelli

Rapid approximate predictions of pressure drop for fully developed, incompressible horizontal gas/liquid flow may be made using the method of Lockhart and Martinelli (Chem. Eng. Prog., 45, 39–48 [1949]). First, the pressure drops that would be expected for each of the two phases as if flowing alone in single-phase flow are calculated. The Lockhart-Martinelli parameter X is defined in terms of the ratio of these pressure drops:

$$X = \left[\frac{(\Delta p/L)_L}{(\Delta p/L)_G}\right]^{1/2}$$

The two-phase pressure drop may be then be estimated from either of the single-phase pressure drops, using;

$$\left(\frac{\Delta p}{L}\right)_{TP} = Y_L\left(\frac{\Delta p}{L}\right)_L$$

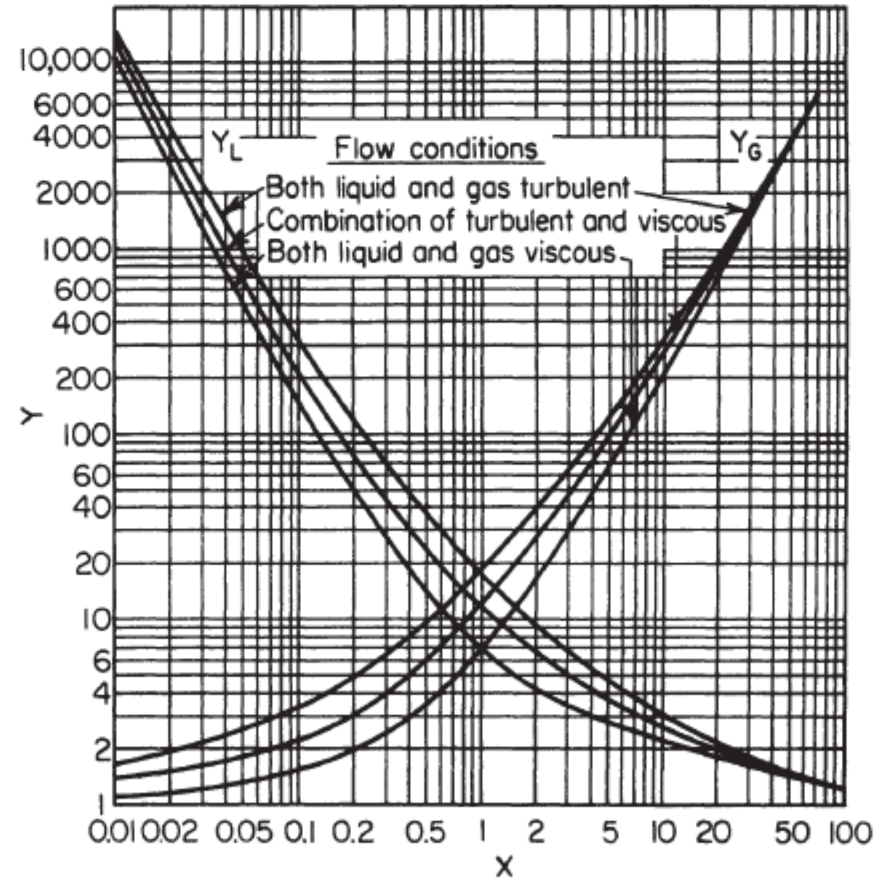
$$\left(\frac{\Delta p}{L}\right)_{TP} = Y_G\left(\frac{\Delta p}{L}\right)_G$$

where Y_L and Y_G are read from the accompanying figure as functions of X. The curve labels refer to the flow regime (laminar or turbulent) found for each of the phases flowing alone. The common turbulent-turbulent case is approximated well by;

$$Y_L = 1 + \frac{20}{X} + \frac{1}{X^2}$$

Multi-Phase Flow
Category 1 – Lockhart and Martinelli

Lockhart and Martinelli correlated pressure drop data from pipes 25 mm (1 in) in diameter or less within about 650 percent. In general, the predictions are high for stratified, wavy, and slug flows and low for annular flow. The correlation can be applied to pipe diameters up to about 0.1 m (4 in) with about the same accuracy. A sample calculation is shown at the end of this module.



Parameters for pressure drop in liquid/gas flow through horizontal pipes. (*Based on Lockhart and Martinelli*, Chem. Engr. Prog., **45**, 39 [*1949*].)

Category 3 - Beggs and Brill

Developed from experimental data from small scale test facility and field data:

The correlation presents different values for liquid holdup for each horizontal flow regime. Initially, the liquid holdup is calculated assuming the pipe is horizontal and then the value is corrected for the actual pipe inclination.

The correlation is modified to include transition zone between segregated and intermittent flow regimes.

A two-phase friction factor, independent of flow regime but a function of holdup is calculated.

Flow Regime Determination

The following variables are used to determine which flow regime would exist if the pipe were horizontal. This flow regime is a correlating parameter and does not reflect the actual flow regime.

Froude Number: $N_{fr} = V_m^2 / (g.d)$

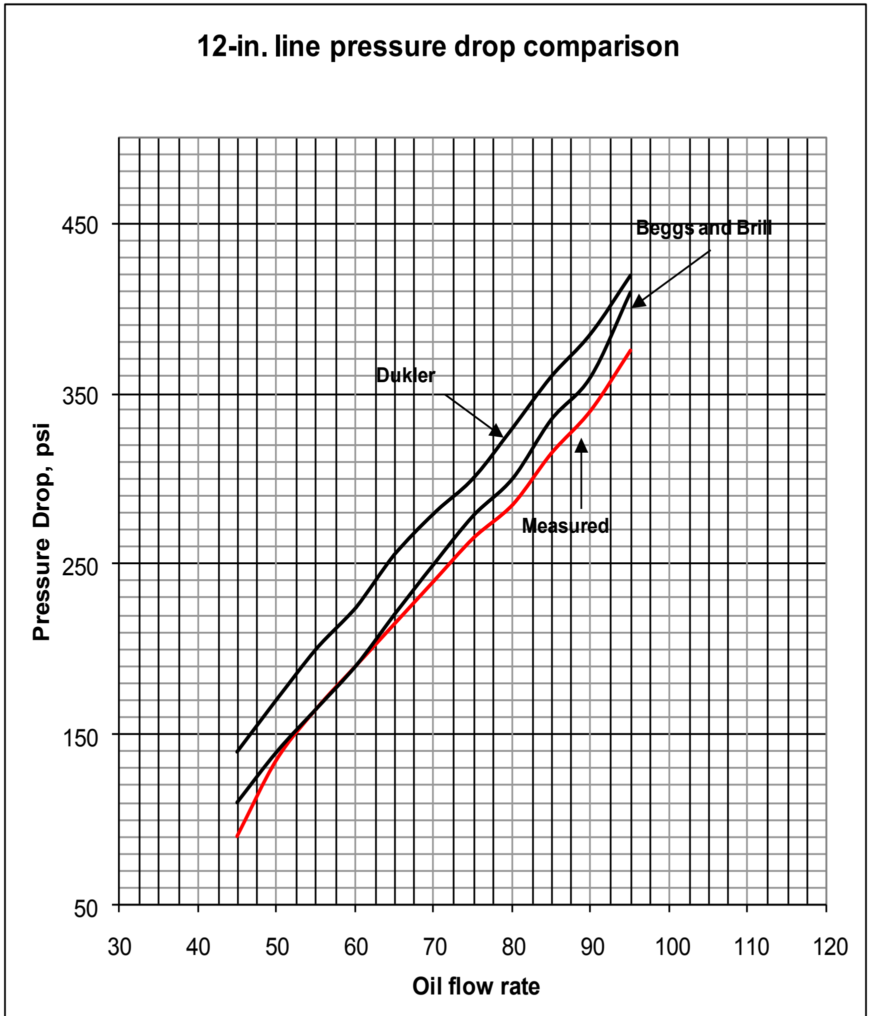
Liquid Velocity Number $N_{Lv} = v_{sl} . (\rho_l . \sigma)^{0.25}$

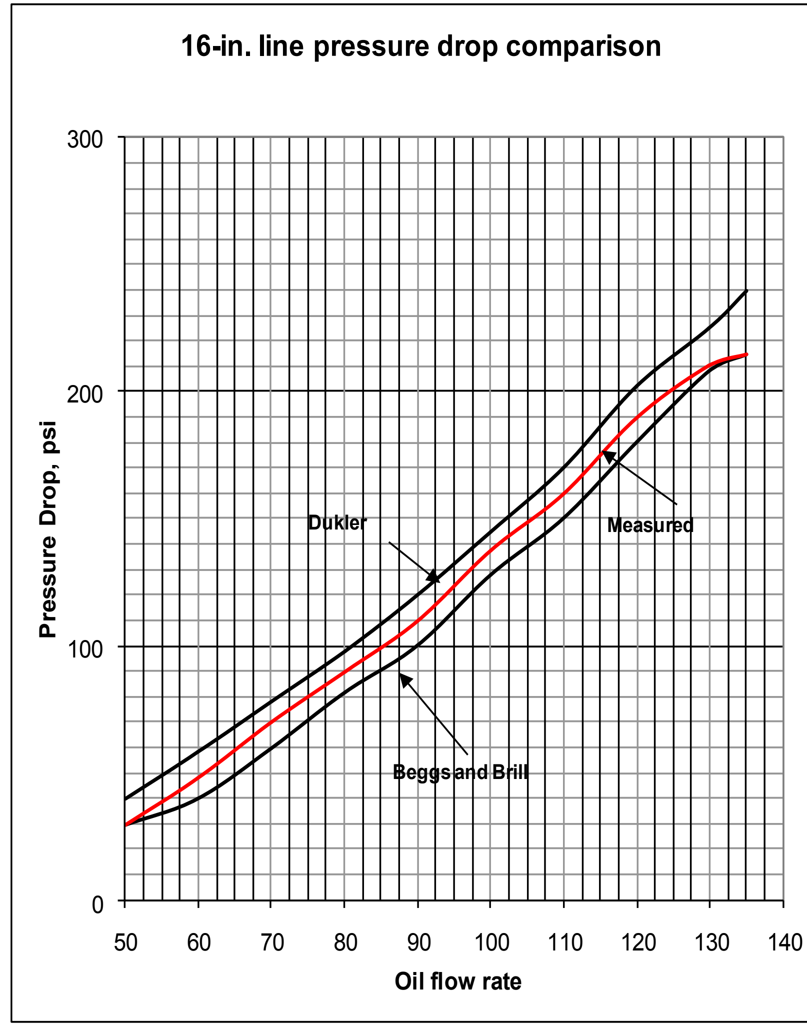
$\lambda_l = v_{sl} / v_m$ and $\lambda_g = v_{sg} / v_m$

A worked example is included at the end of this section – for reference.

Data of Fayed and Otten

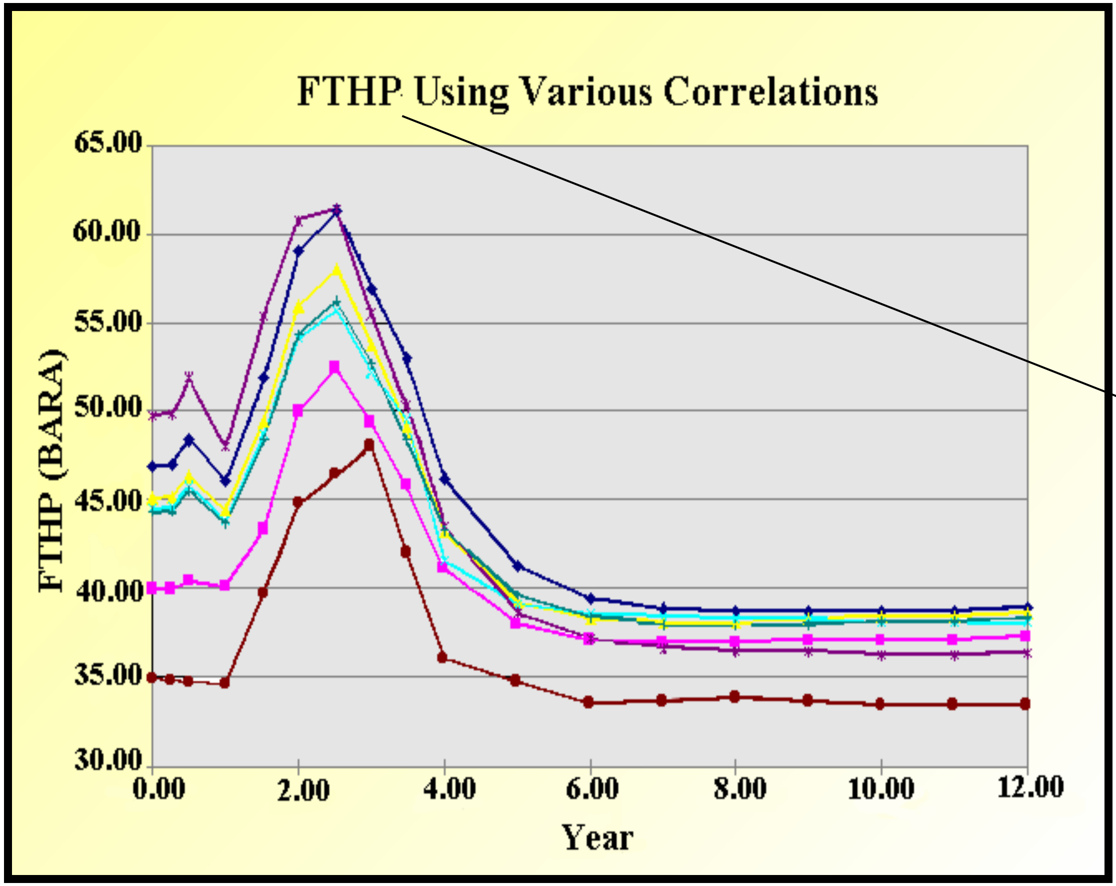
Comparing Measured with Calculated Multiphase Flow, Oil and Gas Journal.



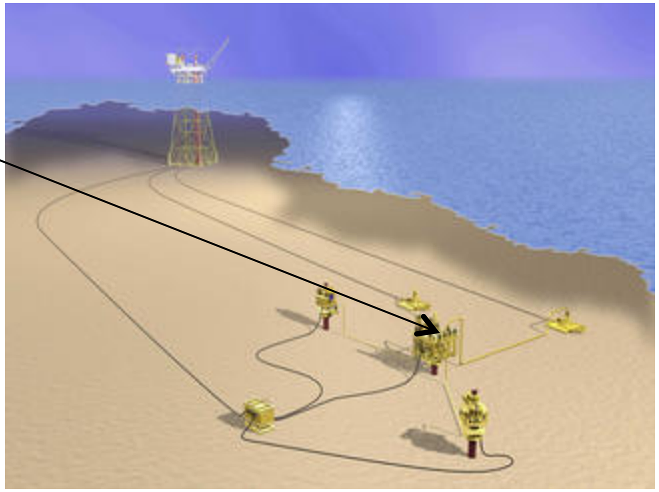


Pressure Loss Correlation - Selection

Basis for hydraulic simulation is same - only change is methodology utlised.

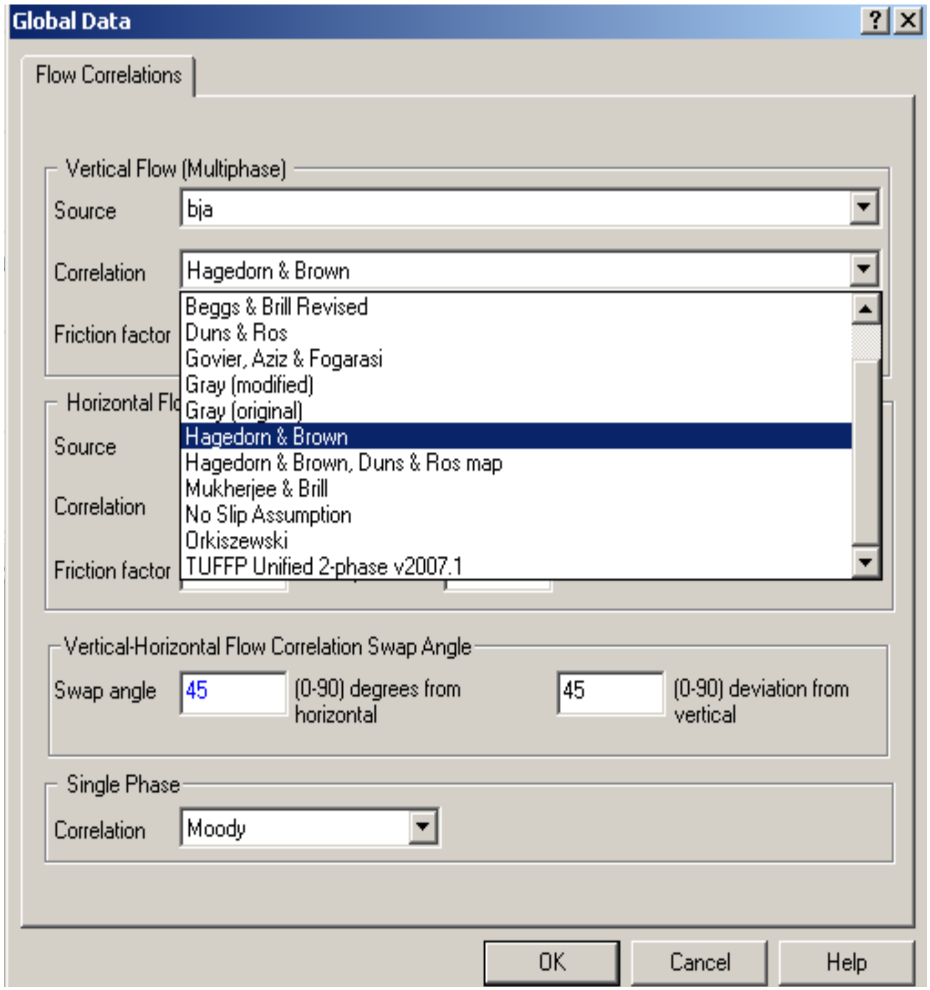


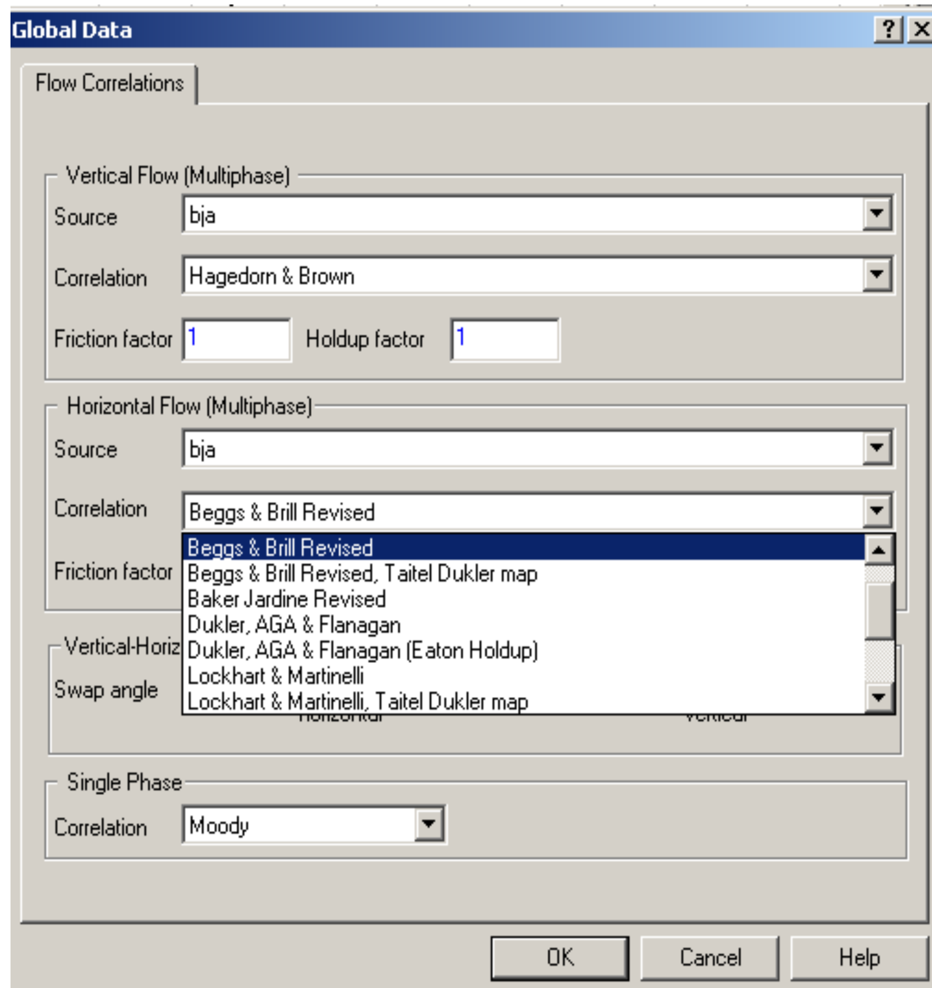
Required FTHP (flowing tubing head pressure) calculated over life field. All information and properties held constant except correlation used for pressure loss predictio.n



Clearly a wide range in outcomes dependent upon methodology selected.

Pressure Loss Correlations available within commercial simulator





OLGAS Model

OLGAS is based on data from the SINTEF two-phase flow laboratory near Trondheim, Norway. The test facilities were designed to operate at conditions that approximated field conditions.

- The test loop was 800m long and of 8” diameter and operated at pressures between 20 and 90 barg.
- Gas superficial velocities of up to 13 m/s, and liquid superficial velocities of up to 4 m/s were obtained.
- Different hydrocarbon liquids were used (naptha, diesel, and lube oil) in order to simulate the range of viscosities and surface tensions. Nitrogen was used as the gas.
- Pipeline inclination angles between 1° were studied in addition to flow up or down a hill section ahead of a 50m high vertical riser.
- Over 10,000 experiments were run on this test loop during an eight year period running in both steady state and transient modes.
- OLGAS considers four flow regimes and uses a unique minimum slip criteria to predict flow regime transitions.

OLGAS Simulator

OLGAS has become an industry accepted tool for the analysis of steady state multi-phase systems. Also offered by the same group is OLGA – a state of the art transient mutli-phase simulator.

OLGA and OLGAS also have OVIP (OLGA® Verification an Improvement Project) . This is a multi-client joint industry research and development program for validation and improvement of OLGA®. The OVIP program is supported by BP, Chevron, ENI, ExxonMobil, Petrobras, Statoil, Total and Shell.

OVIP embodies by far the largest multiphase flow data collection ever assembled. Containing both experimental and field data, including gas/condensate pipelines, lower GOR pipelines, well, and transient data.

Shell test loop in Bacton (UK).

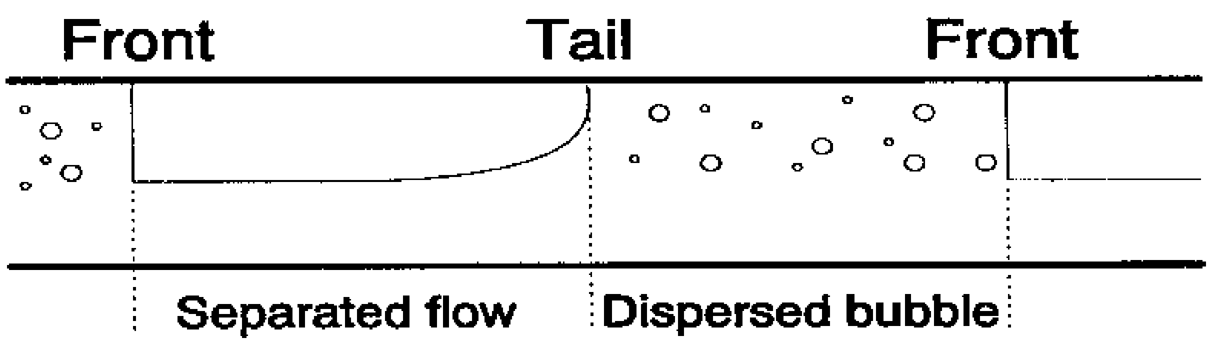
Hydrodynamic Slugging

Hydrodynamic slugs occur spontaneously as a result of steady state operation in the slug flow regime (see flow regime map).

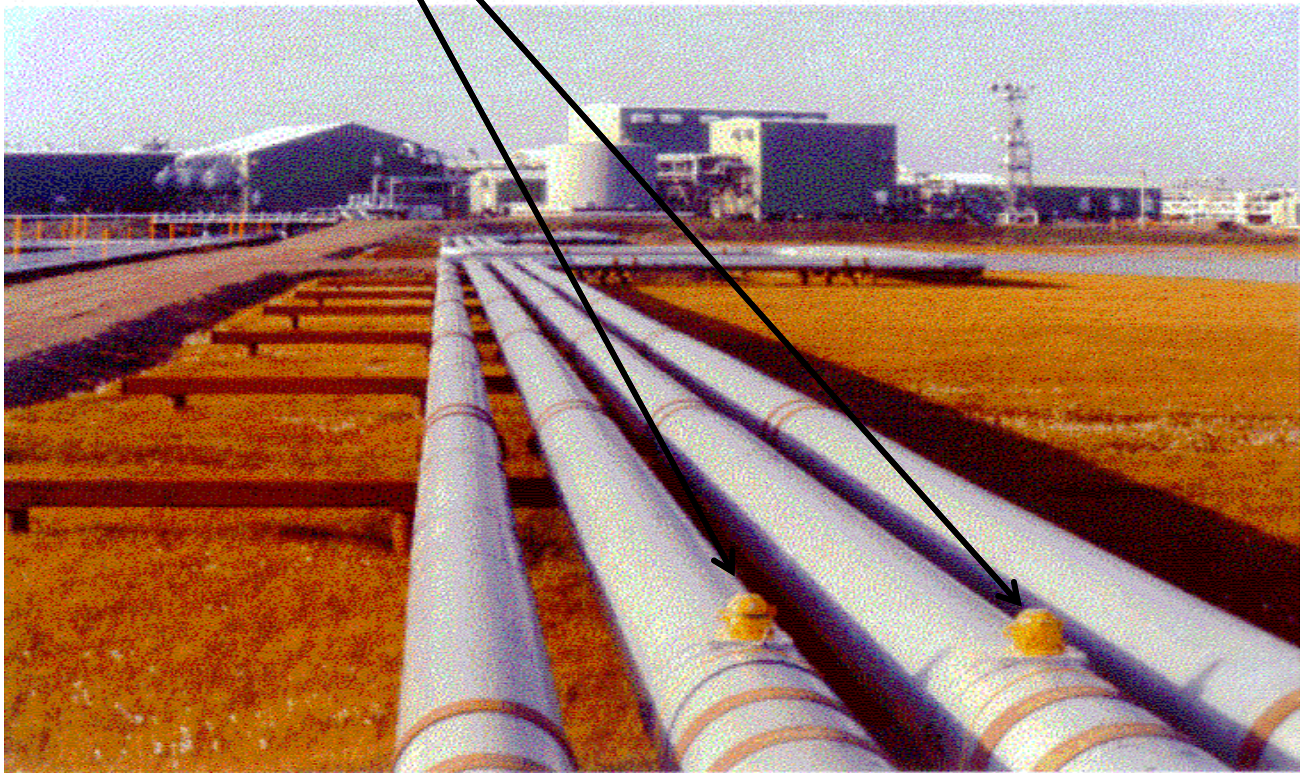
These slugs occur due to an instability of the fluids gas/liquid interface causing liquid waves to reach the top of the pipe.

The slugs are dependent of gas and liquid flowrates, pipe inclination, surface tension, densities and viscosities of the fluid.

Hydrodynamic slugs are typically 30 – 100 pipe diameters and length.



Field Gamma Densitometers Identifying Slug Passage



Gamma Densitometer Traces

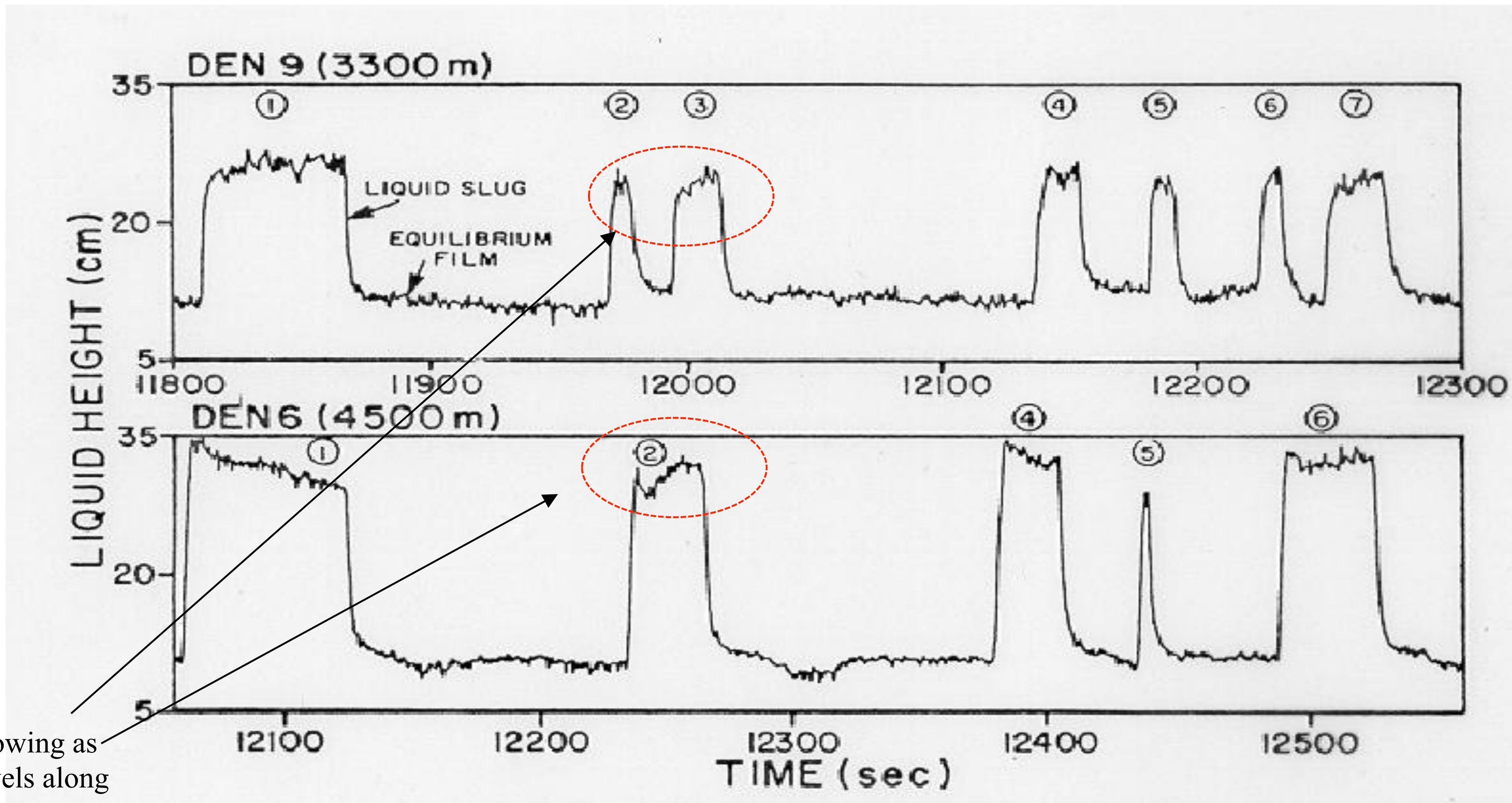


Figure 11 – Densitometer Data for Long Term Slug Growth

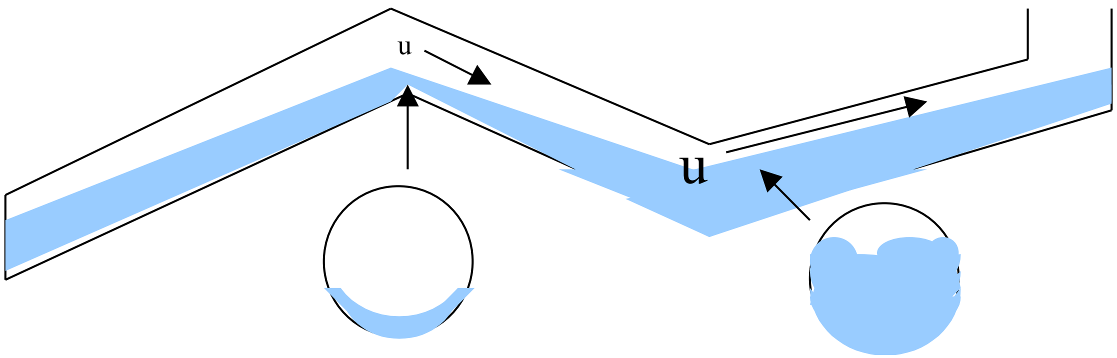
Slugs growing as fluid travels along pipeline.

Terrain Induced Slugging

As the pipe angle changes (up), the liquid hold-up increases reducing the area available for gas flow.

This reduced area causes the gas to accelerate creating waves on the film surface which grow to form slugs.

Slugs grow as they pick up liquid in low spots or join with an adjacent slug.



The empirical Scott, Shoham and Brill correlation takes account of slug growth and can be used as a first approximation.

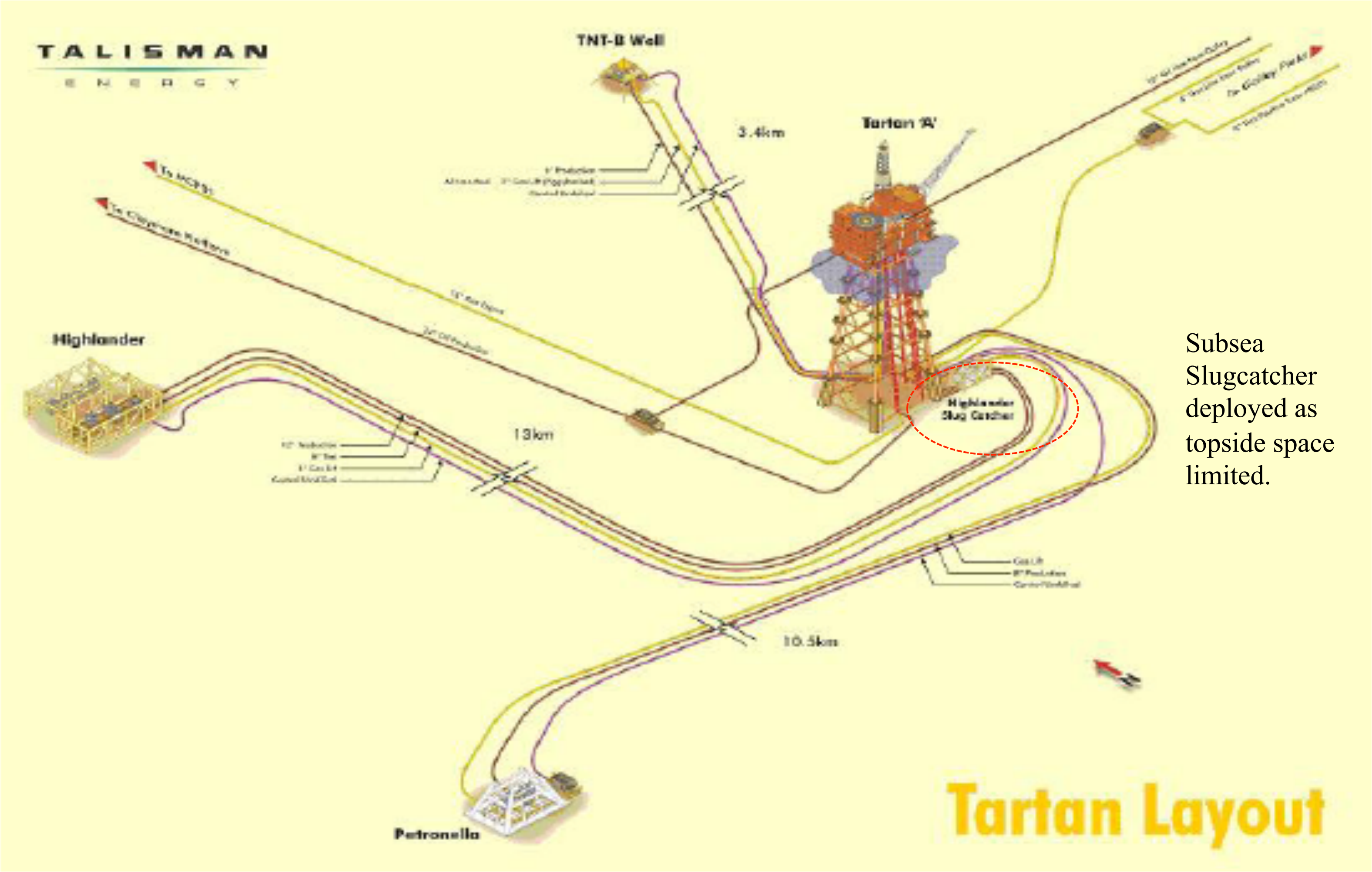
$$\ln(L_m) = -2.663 + 5.441\,[\ln(d)]^{0.5} + 0.059\,[\ln(V_m)]$$

where L_m = mean slug length(ft)

d = pipe diameter (in)

V_m = mixture velocity (ft/sec)

Subsea Slugcatcher

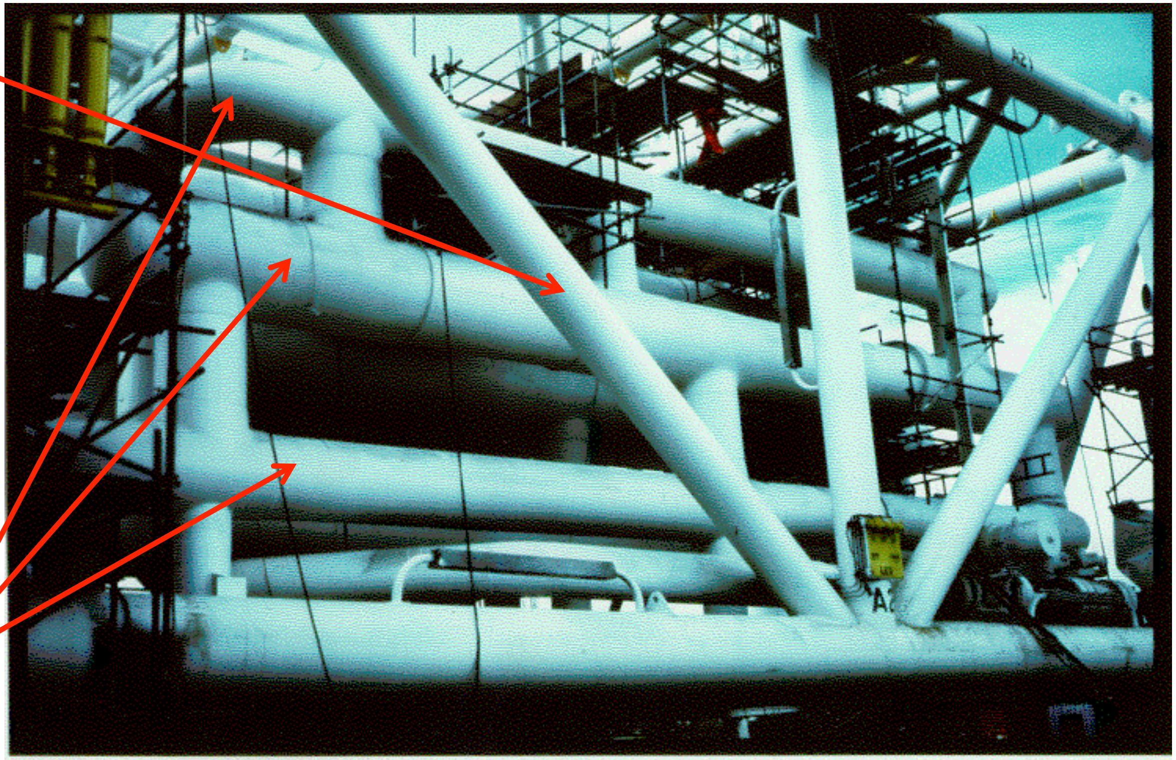


Subsea Slugcatcher deployed as topside space limited.

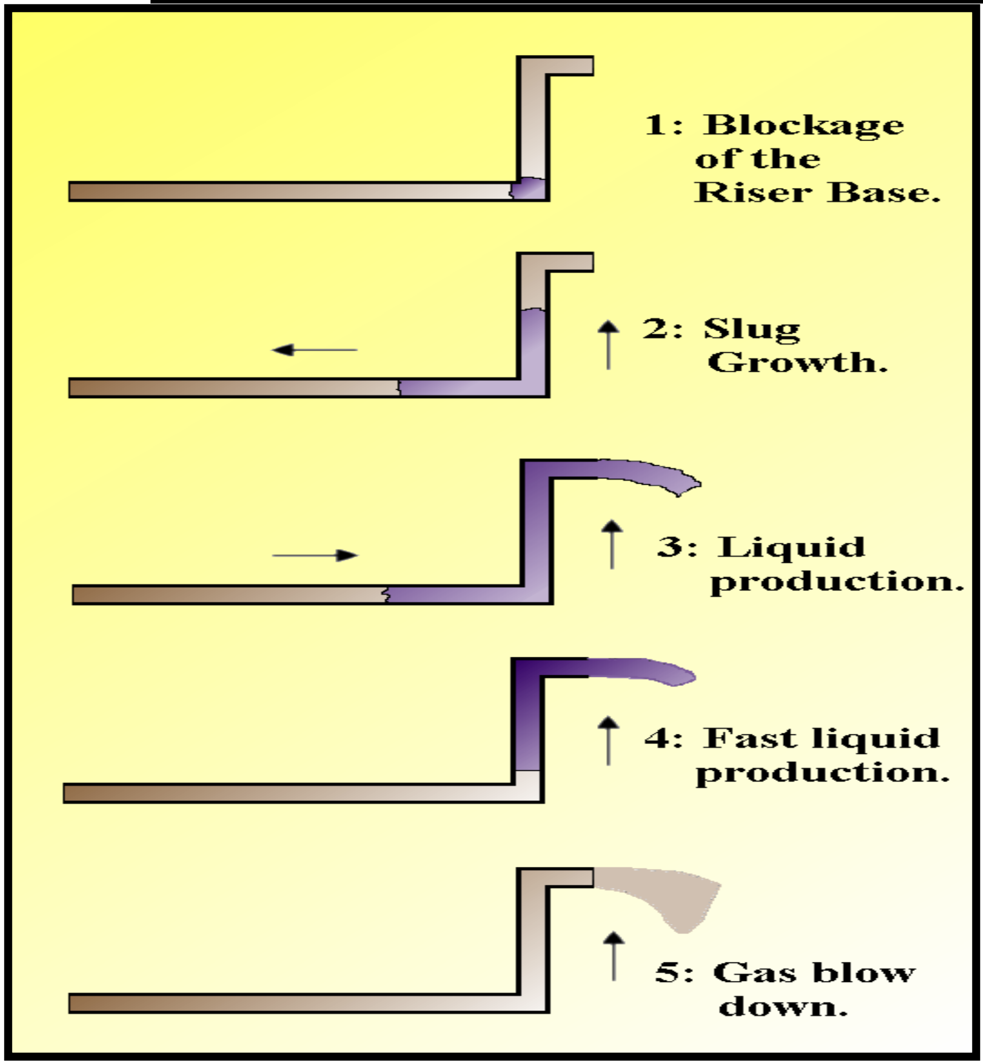
Subsea Slugcatcher

Structural framing

Slug catcher – three horseshoe pipes



Severe Riser Slugging



Severe slugging is an extreme case of terrain induced slugging. The cycle of slugging is described.
Pipeline of downward gradient meets riser base.
Liquid blocks the base of the riser.
Riser fills with liquid if accompanying gas bubble has insufficient pressure to overcome liquid head.
Riser continues to fill.
Once riser is full the backpressure stabilises however the accompanying gas bubble pressure continues to rise
Once the gas bubble pressure is greater than the riser back pressure the riser starts to unpack.
Slowly at first then accelerating as the head in the riser reduces.
Severe slugging produces gas and liquid flow and pressure transients which are very difficult to manage.

Severe Riser Slugging - Mechanism

Severe slugging can occur if the following four conditions are satisfied:

Condition 1 – Dominant Condition

Liquid blockage at the riser base can only occur if;

$(dp/dt)_{riser} > (dp/dt)_{flowline}$

Here the latter pressure gradient is the rate of pressure increase in the pipeline upstream of the riser foot due to gas compression in the pipeline, whereas the former pressure gradient is the rate of pressure increase at the riser base due to an increasing liquid head caused by liquid entering the riser.

The ratio of $(dp/dt)_{flowline} / (dp/dt)_{riser}$ is known as the severe slugging number often labelled $\Pi_{ss.}$

Severe Riser Slugging - Mechanism

For severe slugging to occur $\Pi ss < 1$ (the Boe criterion). This criterion shows that a decreasing GLR (gas liquid ratio) will enhance severe slugging. This can occur at late field life, where water break through will decrease the GLR.

Condition 2 Dominant Condition

The pipeline topography has a low point at the riser foot where liquid blockage may occur.

Condition 3 Less Dominant

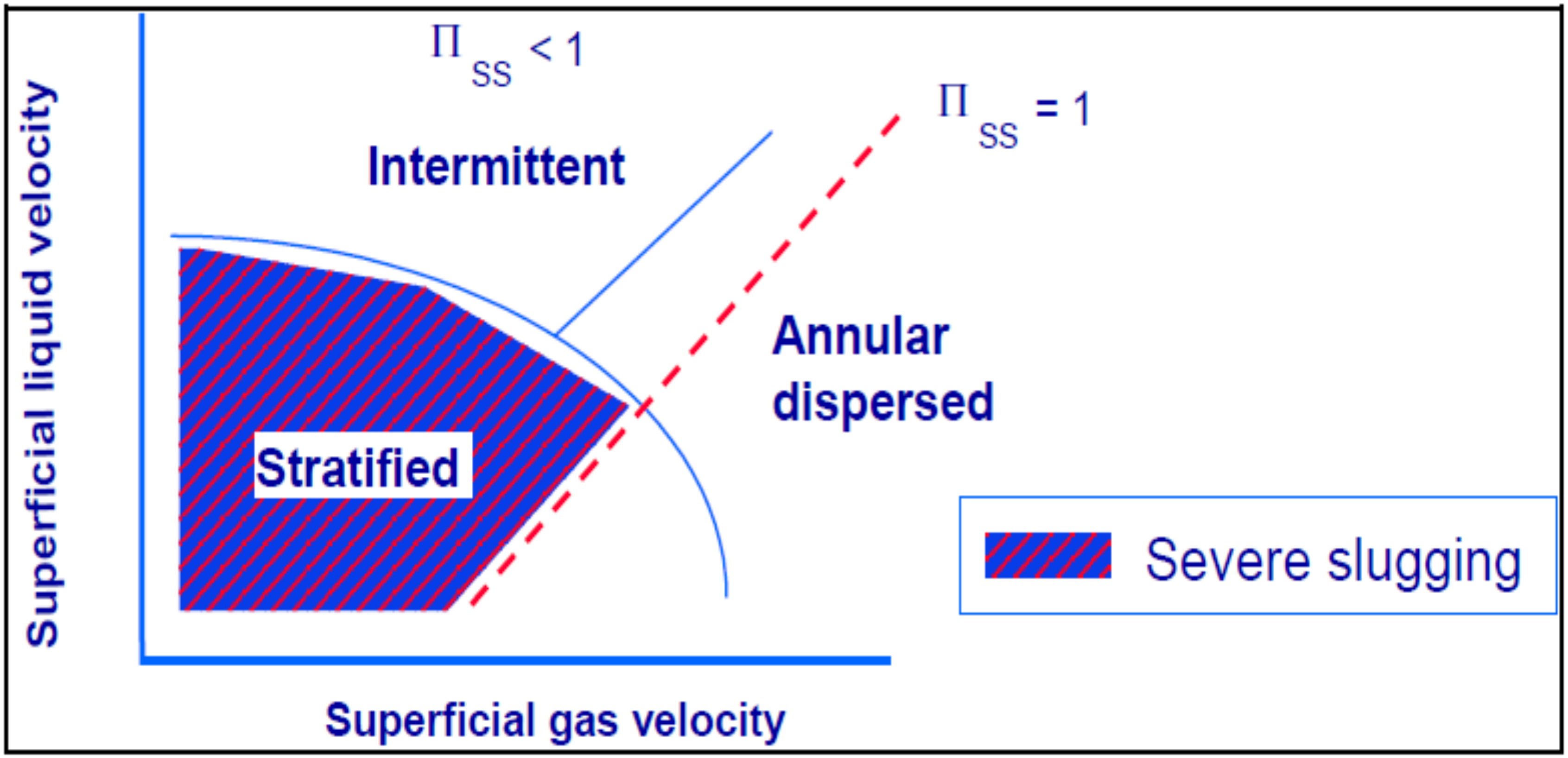
The flow line is operated either in the stratified or annular flow pattern but not in slug flow.

Severe Riser Slugging - Mechanism

The criteria for severe slugging are summarized in the flow pattern map for the pipeline upstream of the riser foot, as depicted in the following. Severe slugging will typically occur at turndown production, and/or at late field life when the riser is in the hydrodynamic slugging regime and the gas production is low.

When severe slugging occurs, more energy is consumed due to the regularly varying gas and liquid velocities in the line. Extra static head loss occurs when the riser is fully filled with liquid. These unsteady effects are not allowed for in the steady state flow simulation programs.

Severe Riser Slugging

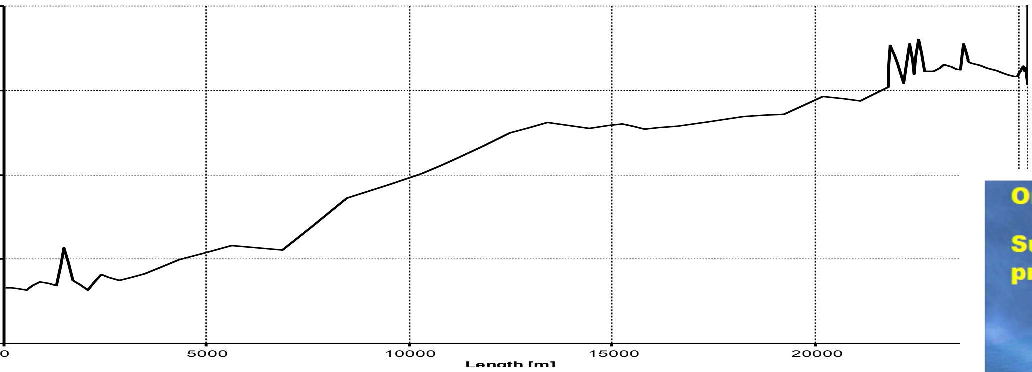


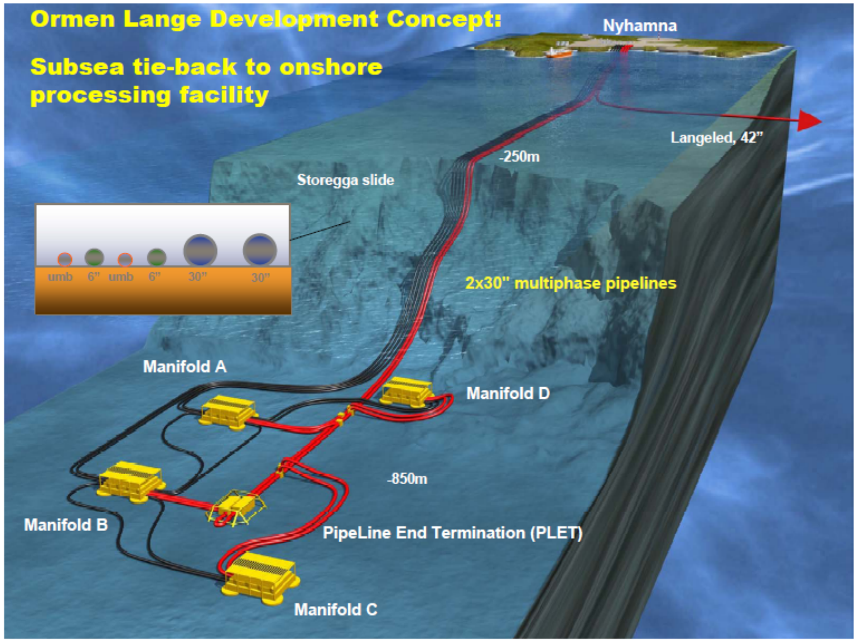
Procedure for establishing likelihood of Severe Riser Slugging

1. Assume all the liquid in the system acts to fill the riser – calculate $(dp/dt)_{riser.}$ Calculate rate at which riser fills in m/s and convert to a pressure rise.
2. Calculate the no slip hold up and, assume no slip exists in the downward sloping portion of the flowline.
3. Calculate the gas volume in the downward sloping line assuming no slip.
4. Calculate the gas volume flow at average pipe line conditions .
5. Calculate $(dp/dt)_{flowline}$ the ideal gas law – rate of pressure rise will be linear with volume flow into gas volume in the sloping line section.
6. Evaluate $(dp/dt)_{flowline} / (dp/dt)_{riser}$ – less than one severe slugging possible.
7. Calculate superficial velocities and check for stratified flow or annular flow.

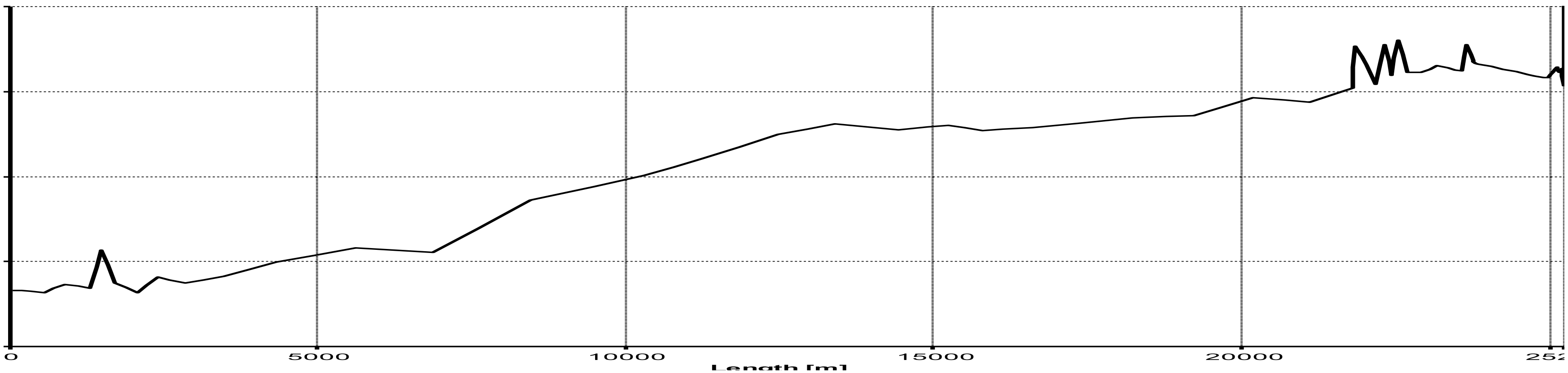
Pipeline Topography

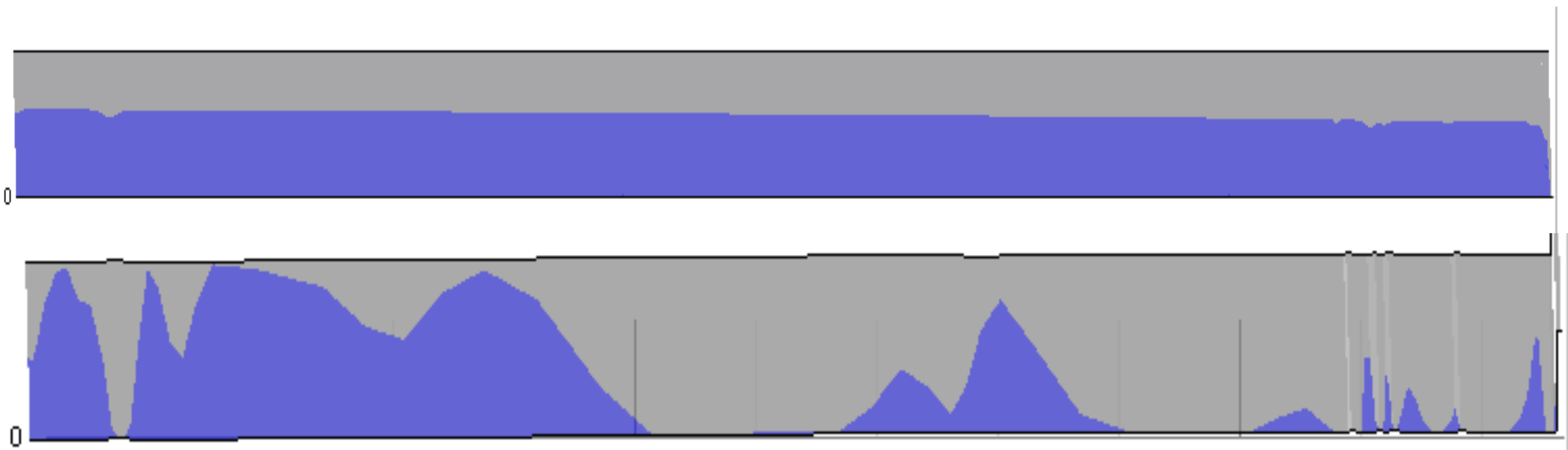
Pipeline topography can have a large influence on slugging characteristics and also on pressure loss. Accurate seabed bathymetry is very important for multi-phase analysis.





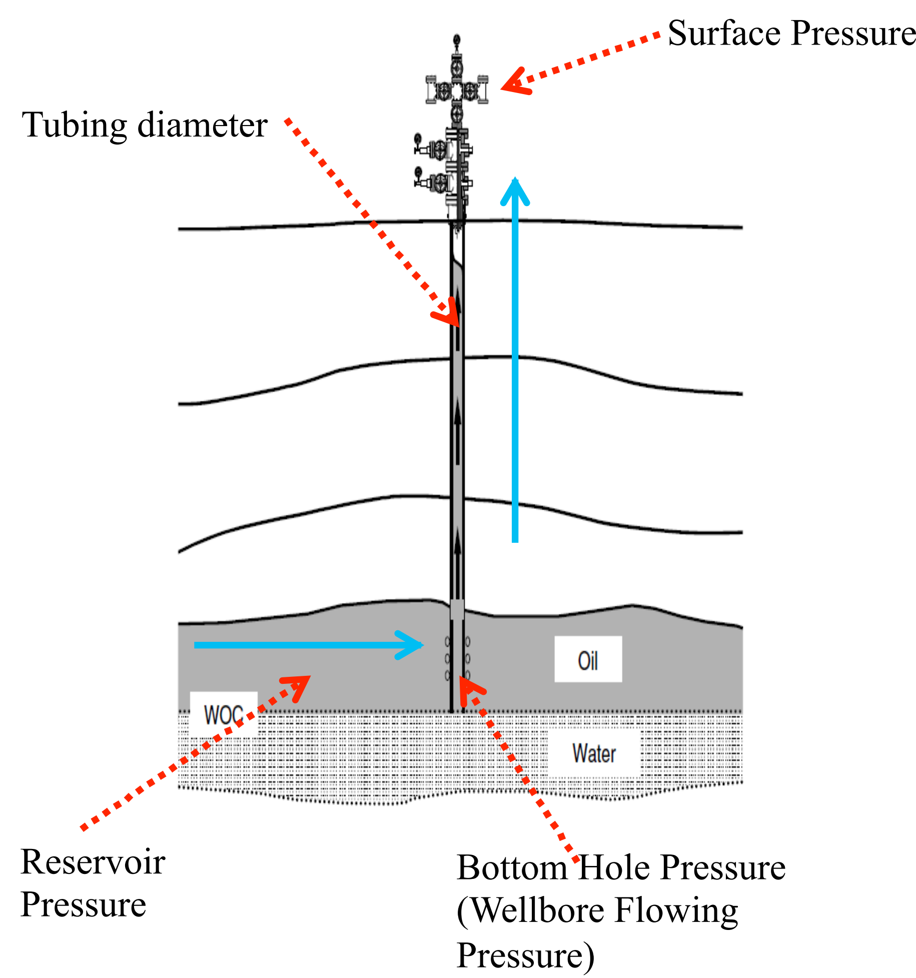
Olga Animations





Coupling of hydrocarbon reservoir and production wells

A key use of fluid mechanics is the prediction of flowrates from producing wells. Here the reservoir is coupled to the well. A stable condition is reached where the rate the reservoir can deliver is matched by the resistance to flow in the well. A convenient way to analyse the reservoir and well behaviour is by the application of gradient or performance curves.



Production Wells - Performance Relationships

The reservoir is characterised by an Inflow Performance Relationship (IPR). This describes the flow through the rock into the well.

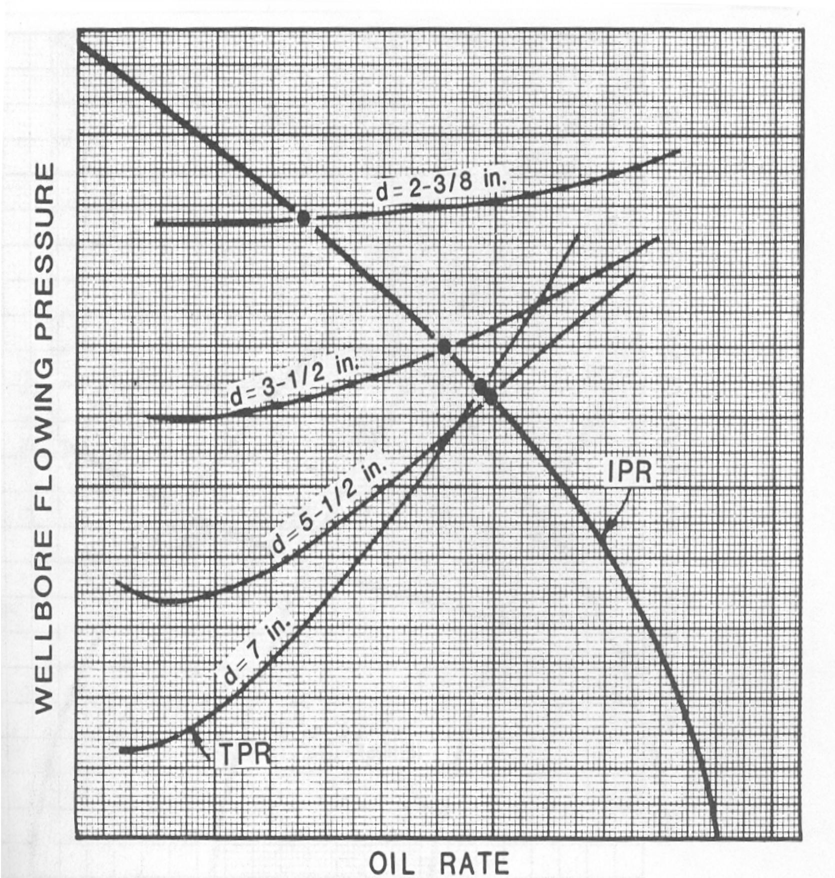
The well is characterised by a Tubing Performance Relationship (TPR). This describes the fluid mechanics in the well. This can change from single phase to two-phase as the fluid pressures reduces moving up the well. The fluid mechanics therefore becomes very complex.

The analysis is usually undertaken on specialist software.

The intersection of the IPR and TPR curves determines the rate of stable flow that can be expected from a particular well.

Gradient Curves – Variable Sensitivity

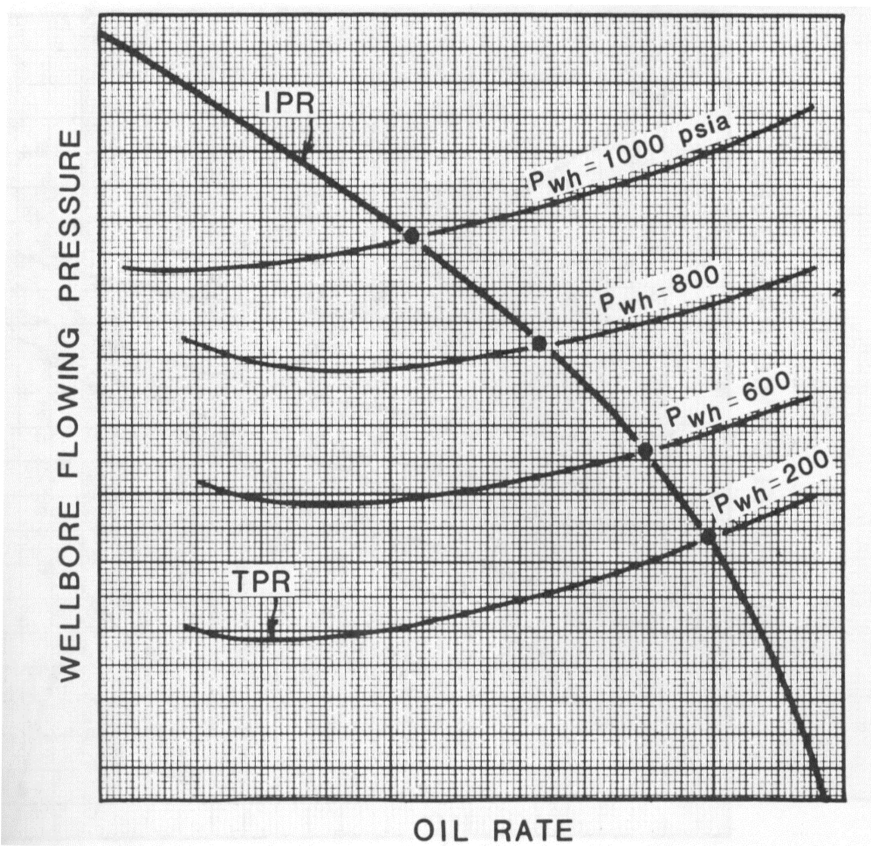
Diameter



- The effect of changing tubing diameter is shown for a mixture of liquid and vapour in a well.
- Increasing diameter increases the production rate until an optimal diameter is reached. Increasing the diameter further results in a rate reduction.
- This is a consequence of the two components of pressure drop – friction and elevation. At higher diameters the liquid tends to ‘slump’ within the well resulting in higher mixture densities. Hence elevation/hydrostatic head increase is higher than the drop in friction losses.
- This effect would not be evident with single phase flow.

Gradient Curves – Variable Sensitivity

Surface Pressure



The effect of varying surface pressure is shown for a mixture of liquid and vapour in a well.

Reducing surface pressure increases the production rate .

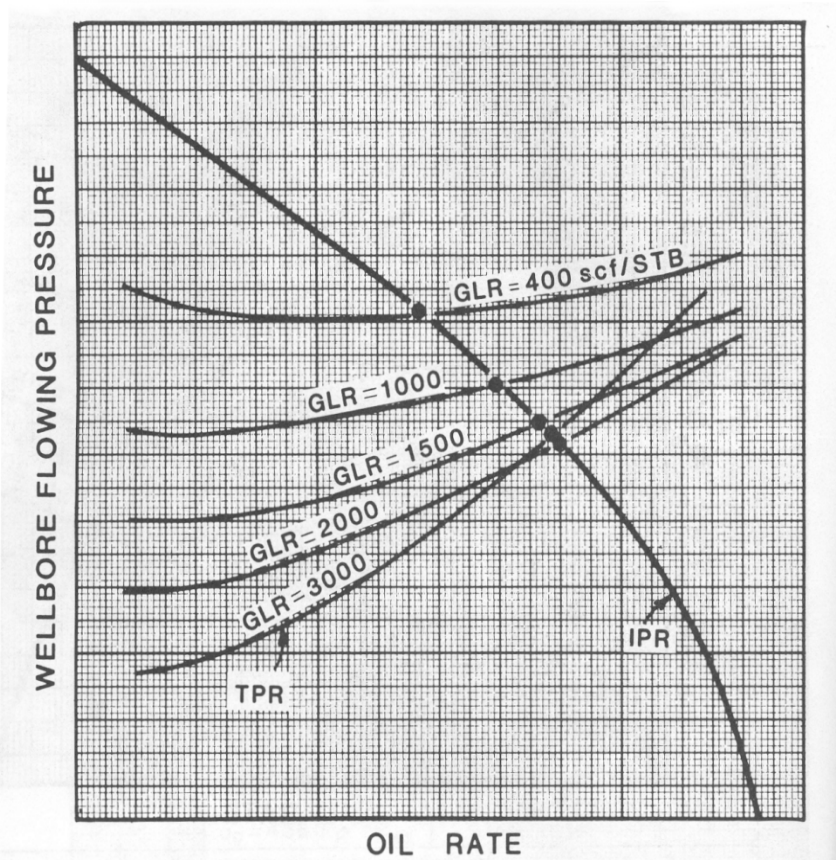
As the surface pressure is reduced the back pressure reduces hence flow naturally increases.

With two phase flow in the well mixture velocities will increase and mixture density reduce as the surface pressures is dropped.

As the surface pressure drops gas volumes at surface increase resulting in large compression requirements.

Gradient Curves – Variable Sensitivity

Gas Liquid Ratio



The effect of varying gas liquid ratio is shown for a mixture of liquid and vapour in a well.

The gas liquid ratio could be increased by injecting gas into the base of a well. This is known as gas lift.

As can be seen there is an optimal GLR where further increases result in a drop off in production rate.

The explanation for the optimal GLR relates to the two pressure loss components of friction and elevation.

Whilst injecting gas will increase the fluid velocity thus increasing the frictional losses, the reduction in mixture density and elevation losses is more significant.

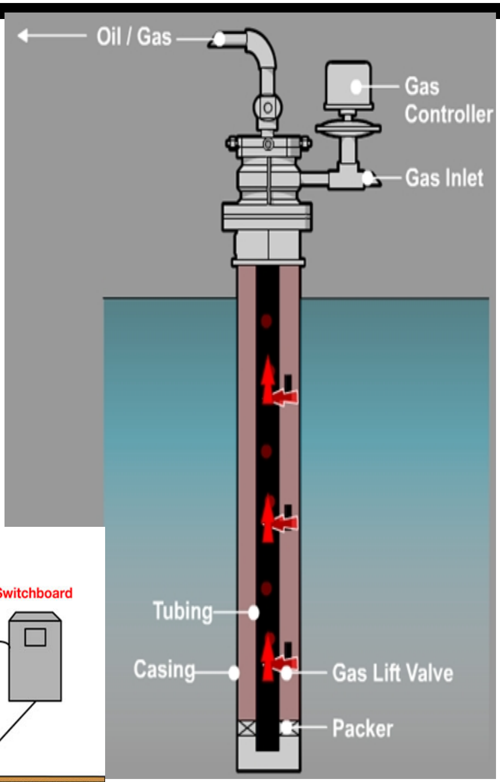
However there comes a point where friction losses start to dominate and the opposite holds.

Artificial Lift

Pressure losses in the production tubing due to friction and elevation can be overcome using artificial lift

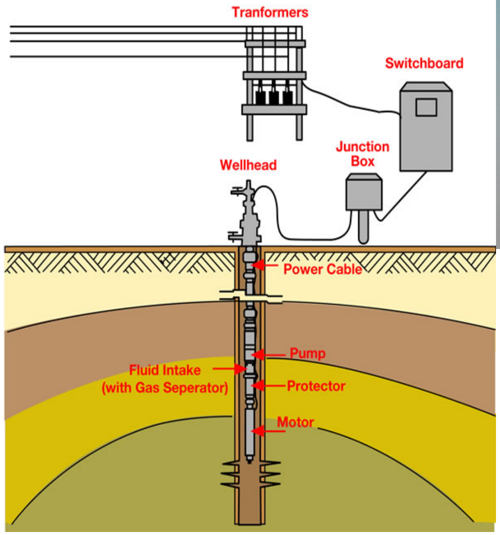
Gas Lift

- Gas is injected into the lower part of the production tubing and mixed with reservoir fluids, reducing the pressure gradient and lowering the backpressure on the reservoir.



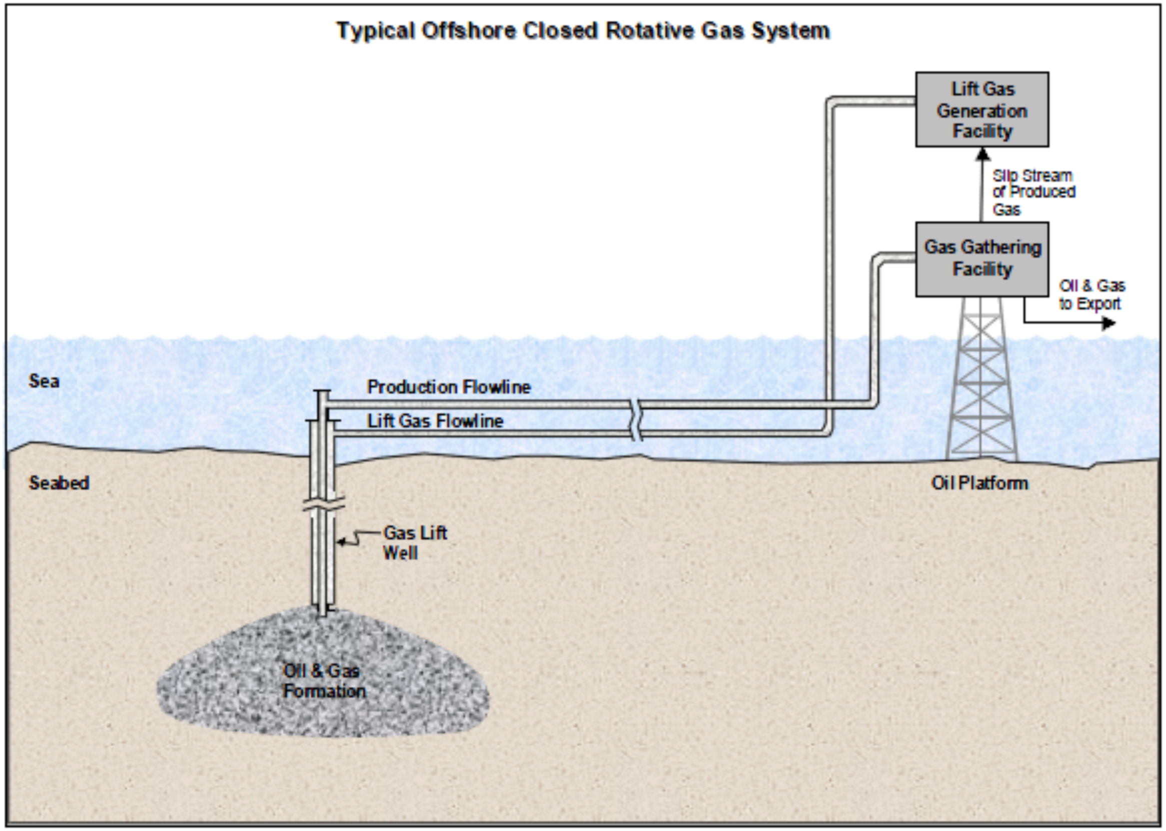
Downhole Pump

- Installing a pump at the bottom of a tubing string creates an artificial lifting capacity and increases the available pressure to flow up the tubing. The pump adds a controlled amount of pressure to the IPR thereby sustaining flow at higher than the natural rate
- Pump types - Electric submersible, hydraulic - turbine and jet



Gas Lift

Lift gas is delivered from the receiving facility. The lift gas is introduced into the produced fluids. The gas compression facility on the host has to handle returning lift gas and the gas associated with the produced oil.



Pipeline Lay Barge

Sections of pipe welded together on ship.

Pipeline Lay Barge

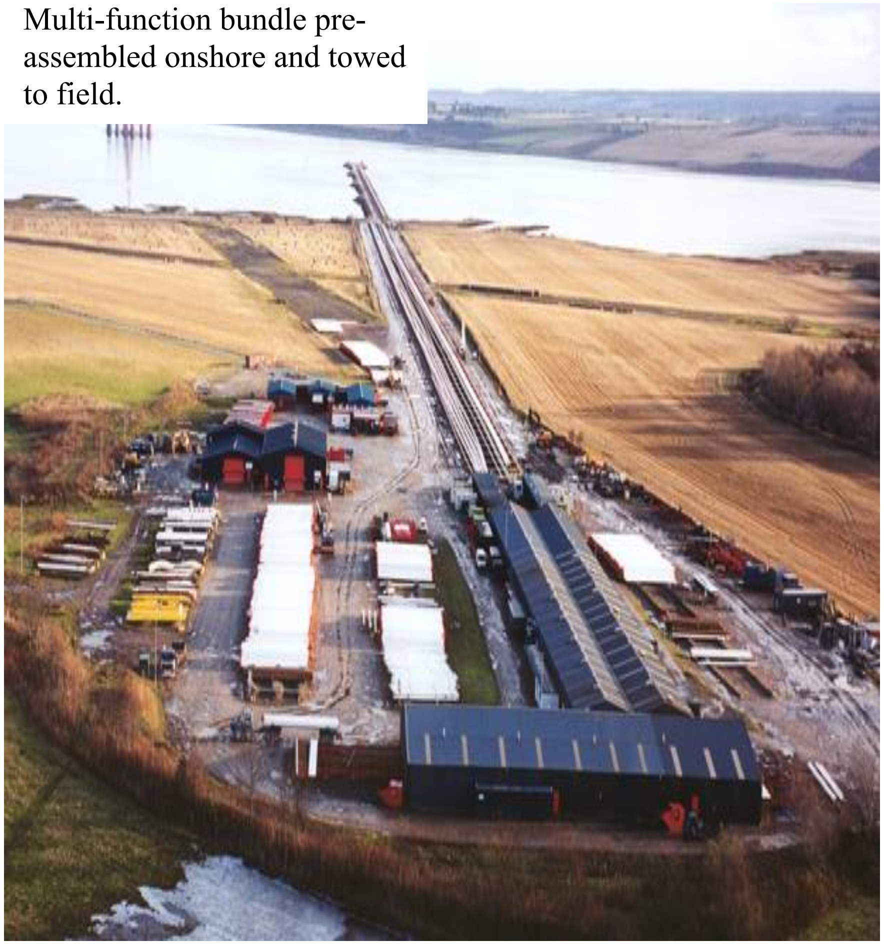
Solitaire, the largest pipelay vessel in the world

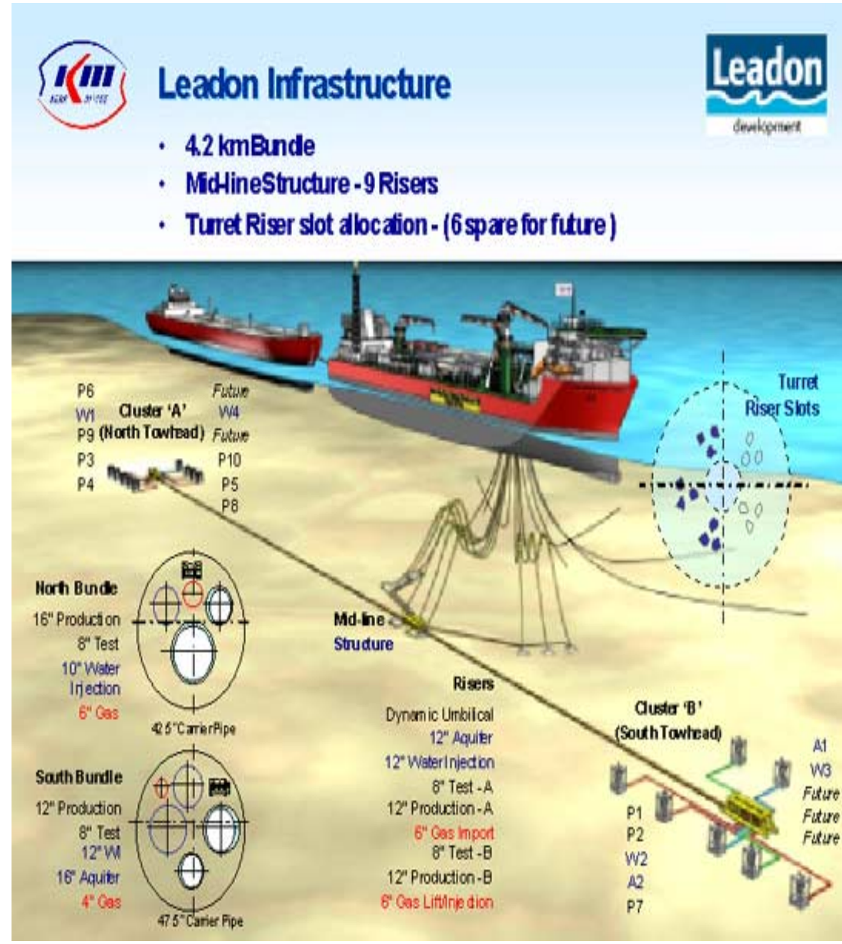
- **Length overall (incl. stinger)**
  397 m (1,302 ft)
- **Length overall (excl. stinger)**
  300 m (984 ft)
- **Length between perpendiculars**
  249 m (817 ft)
- **Breadth**
  41 m (135 ft)
- **Maximum speed**
  13 knots
- **Accommodation**
  420 persons
- **Total installed power**
  51,500 kW
- **Dynamic positioning system**
  LR DP (AAA), fully redundant
  Kongsberg K-Pos DP-22, K-Pos DP-12
  and 3 x cJoy system
- **Deck cranes**
  2 x Pipe transfer cranes of
  (77 kips) at 33 m (108 ft)
  1 x Special purpose crane of
  300 t (661 kips) at 17 m
  1 x PLET installation frame of
  400 t (882 kips)
- **Work stations**
  2 x Double joint factories, each with 3
  welding stations and 1 NDT station
  Main firing line with 5 welding stations
  for double joints, 1 NDT station and 4
  coating stations
- **Tensioner capacity**
  3 x 350 t (3 x 772 kips) at 30 m/min
  (98 ft/min)
- **Pipe diameters**
  From 2" to 60" OD
- **Pipe hold capacity**
  22,000 t

Pipeline Reel Barge

Pipeline Bundles

Multi-function bundle pre-assembled onshore and towed to field.





Key learnings

1. Superficial and mixture velocity
2. Hold – up and phase slippage
3. Flow patterns/regimes – flow regime determination
4. Slug production from pipeline sphering
5. Multi-phase pressure drop methods
6. Slugging flow – form and impact on receiving facility
7. Severe slugging mechanism
8. Severe slug criterion, Π_{ss}
9. Reservoir/Well Characteristics

Taitel –Dukler Flow Regime – Worked Example

Gas mass flux/velocity, G_g = 5 kg/m²s

Liquid " , G_L = 495 kg/m²s

Gas viscosity , μ_g = 1×10^{-5} Pa.s

Liquid viscosity , μ_L = 2×10^{-3} Pa.s

Gas density , ρ_g = 20 kg/m³

Liquid density , ρ_L = 817 kg/m³

Pipe diameter , D = 0.19 m

Pipe angle , θ = 0°

Pipe friction coefficient = $0.0791 \times (N_{Re})^{-0.25}$ (¼ × moody friction factor)

Taitel –Dukler Flow Regime – Worked Example

Calculate 2.

Liquid Reynolds Number, $N_{Re_L} = \frac{\rho . v . D}{\mu} = \frac{G_L D}{\mu_L}$

$\rho . v = kg/m^3 \times \frac{m}{s} = \frac{kg}{m^2 s}$ = mass flux.

$= \frac{495 \times 0.19}{2 \times 10^{-3}}$

$= 47025$

friction coefficient, $C_f = 0.0791 (N_{Re_L})^{-0.25}$

$= 0.0791 \times (47025)^{-0.25}$

$= 0.0054$

Liquid pressure gradient $\left(\frac{dP}{dz}\right)_L = \frac{2 C_f . \rho . v^2}{D}$, $G^2 = \rho^2 v^2$

$\rho v^2 = \frac{G^2}{\rho}$

Taitel –Dukler Flow Regime – Worked Example

3.

$$\left(\frac{dP}{dz}\right)_L = \frac{2\,C_f\,G_L^2}{\rho_L\,D}$$

$$= \frac{2 \times 0.0054 \times 495^2}{817 \times 0.19}$$

$$= 17.0 \ Pa/M$$

Gas Reynolds Number, $N_{Re_g} = \frac{G_g\,D}{\mu_g} = \frac{5 \times 0.19}{1 \times 10^{-5}}$

$$= 95{,}000$$

friction coefficient $= 0.0791 \times (N_{Re_g})^{-0.25}$

$$= 0.0045$$

Gas pressure gradient, $\left(\frac{dP}{dz}\right)_g = \frac{2\,C_f\,G_g^2}{\rho_g\,D}$

$$= 0.059 \, Pa/M.$$

Taitel –Dukler Flow Regime – Worked Example

4

Lockhart Martinelli Parameter, $X = \left[\left(\frac{dP}{dz}\right)_L \Big/ \left(\frac{dP}{dz}\right)_g \right]^{0.5}$

$= 17.0$

Gas phase Froude number, $F = \sqrt{\frac{\rho_g}{\rho_L - \rho_g}} \cdot \frac{j_g}{\sqrt{D \cdot g \cdot \cos\theta}}$

j_g, superficial velocity $(V_{sg}) = \frac{G_g}{\rho_g} = \frac{5}{20} = 0.25 m/s$

$F = \sqrt{\frac{20}{817-20}} \times \frac{0.25}{\sqrt{0.19 \times 9.81 \times \cos(0)}}$

$F = 0.0289$

Taitel –Dukler Flow Regime – Worked Example

5.

T Parameter, $T = \left[\dfrac{(dP/dz)_L}{(\rho_L - \rho_g) \cdot g \cdot \cos\theta} \right]^{1/2}$

$= \left[\dfrac{17.0}{(817 - 20) \cdot 9.81 \cdot \cos(0)} \right]^{1/2}$

$= 0.0466$

K Parameter, $K = F \cdot \left(\dfrac{D \cdot j_f}{\nu_f} \right)^{1/2}$

$(f = L)$

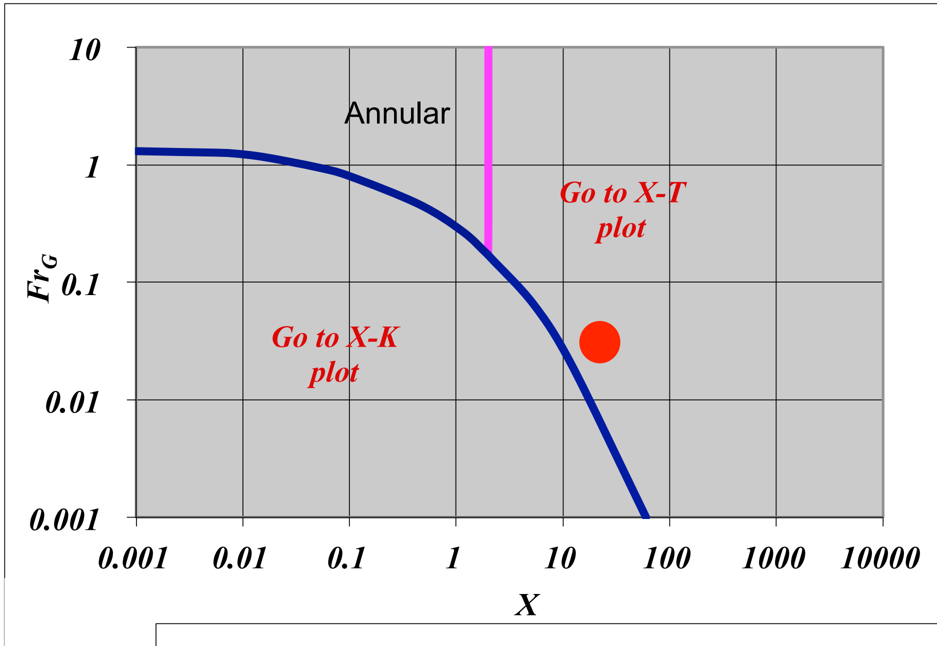
$\nu_f = \mu_L / \rho_L$

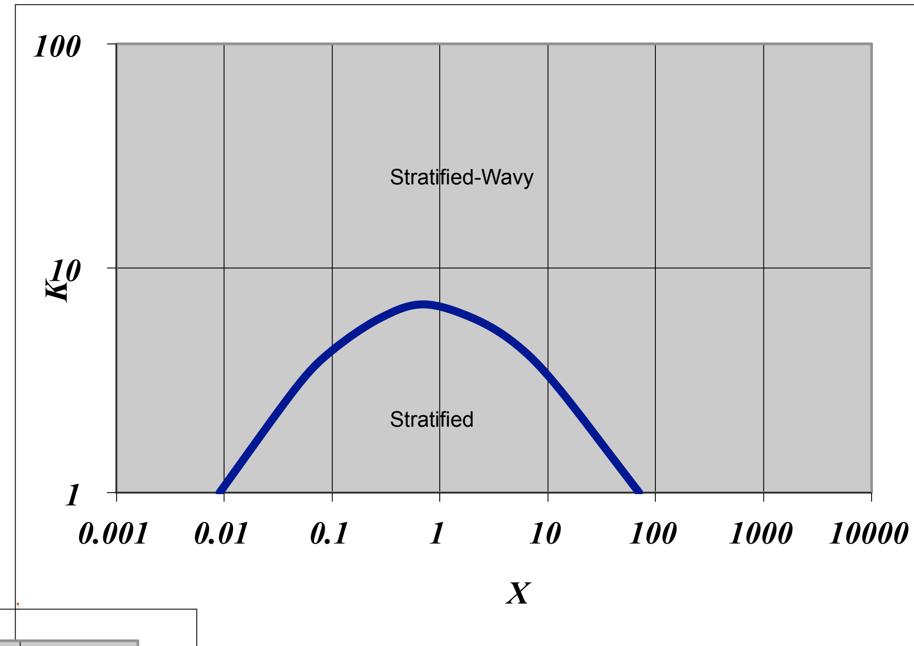
$= F \cdot \left(D \cdot \dfrac{G_L}{\cancel{\rho_L}} \cdot \dfrac{\rho_L}{\mu_L} \right)^{1/2}$

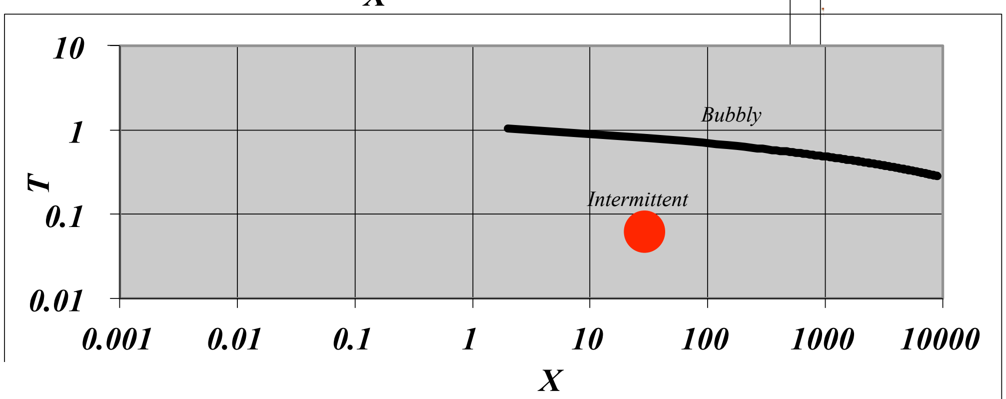
$= F \cdot \left(N_{RE_L} \right)^{1/2}$

$= 6.26$

Taitel –Dukler Flow Regime – Worked Example

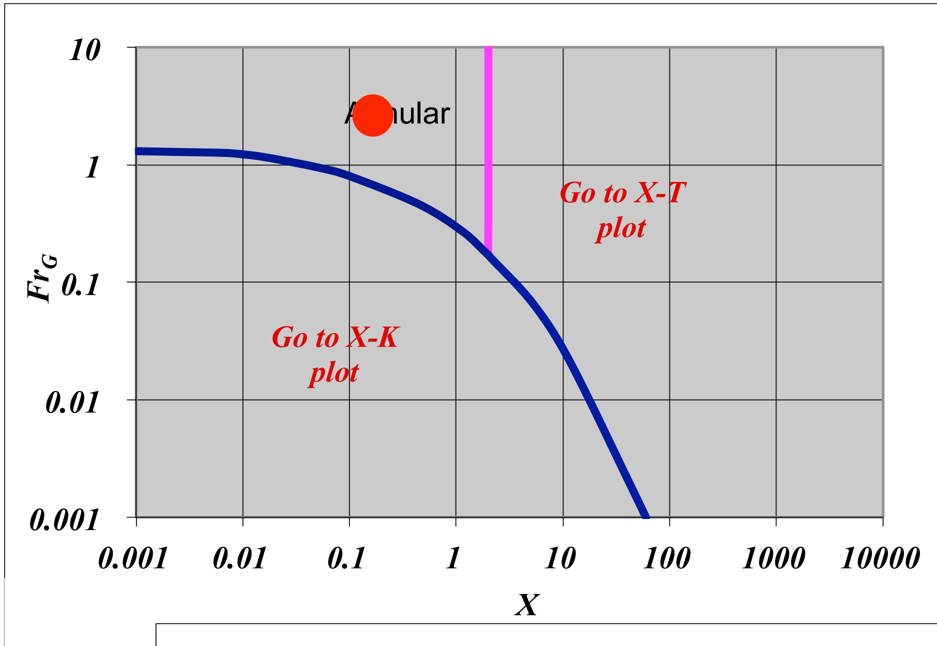


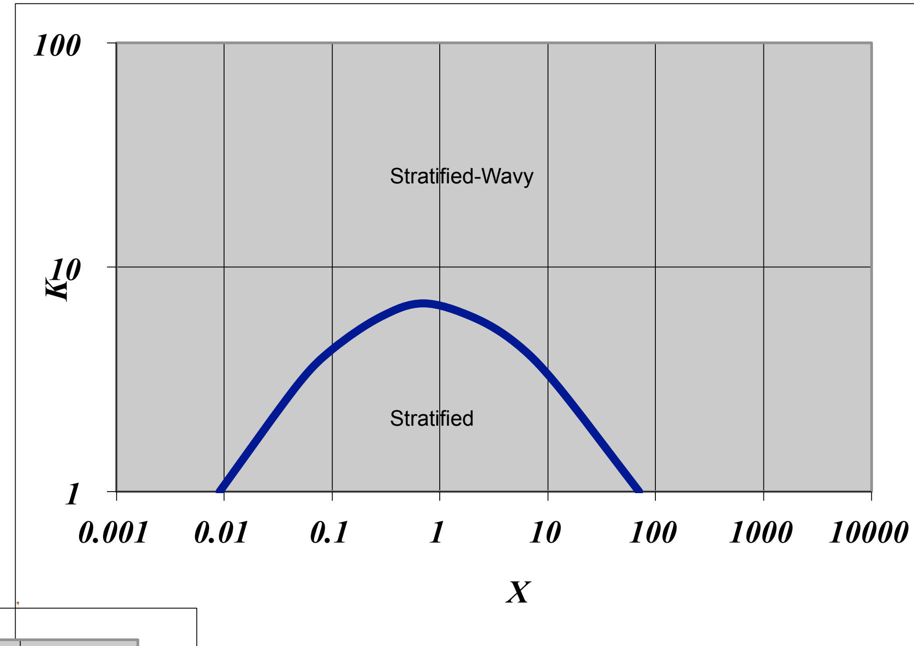


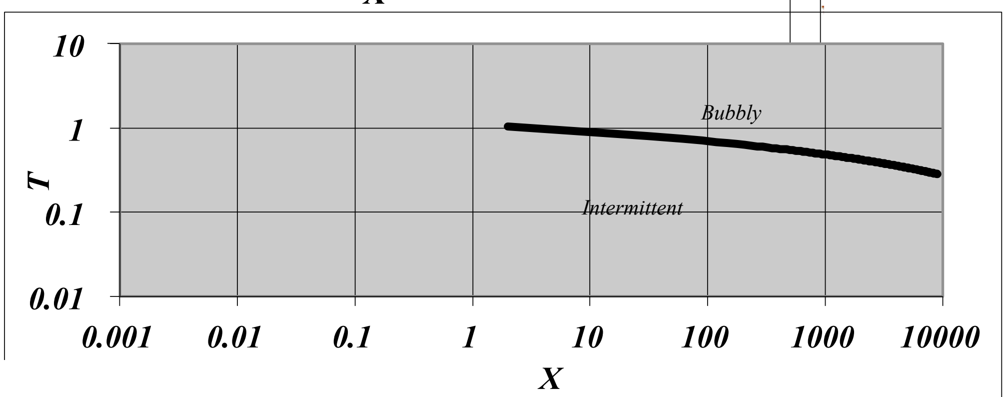


Intermittent or Slug Flow

Repeat with x = 0.5







Annular flow

Hold up – Eaton Method

Eaton Holdup Example

150 mm I.D. pipeline

1200 m long

$\mu_L = 0.02$ Pa.s. liquid viscosity

$Q_L = 17\ m^3/hr$ liquid flow

$Q_G = 425\ m^3/hr$ vapour flow

$\rho_L = 880\ kg/m^3$ liquid density

$\rho_g = 20.8\ kg/m^3$

$\sigma = 1.5 \times 10^{-6}\ N/m$

$$\left.\begin{array}{l} P_{inlet} = 2800\ kPa \\ P_{out} = 2400\ kPa \end{array}\right\} P_{AVE} = 2600\ kPa$$

$P_b = 101.6\ kPa$

Hold up – Eaton Method

Calculate superficial velocities

$$\text{Pipe area} = \pi \times \frac{0.15^2}{4} = 0.0177\ m^2$$

$$V_{sg} = \frac{425}{0.0177 \times 3600} = 6.67\ m/s$$ gas superficial velocity

$$V_{sL} = \frac{17}{0.0177 \times 3600} = 0.27\ m/s$$ liquid superficial velocity

Calculate dimensionless groups

$$N_{Lv} = 0.0565\ V_{sL}\left(\frac{\rho_L}{\sigma}\right)^{0.25} = 0.0565 \times 0.27 \times \left(\frac{880}{1.5 \times 10^{-6}}\right)^{0.25} = 2.3$$

$$N_{gv} = 0.0565\ V_{sg}\left(\frac{\rho_L}{\sigma}\right)^{0.25} = 57$$

Hold up – Eaton Method

$$N_d = 0.00003134\, d \left(\frac{\rho_L}{\sigma}\right)^{0.5} = 116$$

$$N_L = 0.001769\, \mu_L \left(\frac{1}{\rho_L \sigma^3}\right)^{0.25} = 0.15$$

Calculate Eaton's abscissa

$$N_E = \frac{1.84 \left(N_{Lv}\right)^{0.575} \cdot \left(\frac{P_{avg}}{P_b}\right)^{0.05} \cdot \left(N_L\right)^{0.1}}{N_{gv} \cdot \left(N_d\right)^{0.0277}}$$

$$= \frac{1.84 \times 2.3^{0.575} \times \left(\frac{2600}{101.6}\right)^{0.05} \times 0.15^{0.1}}{57 \times 116^{0.0277}}$$

$$= 0.044$$

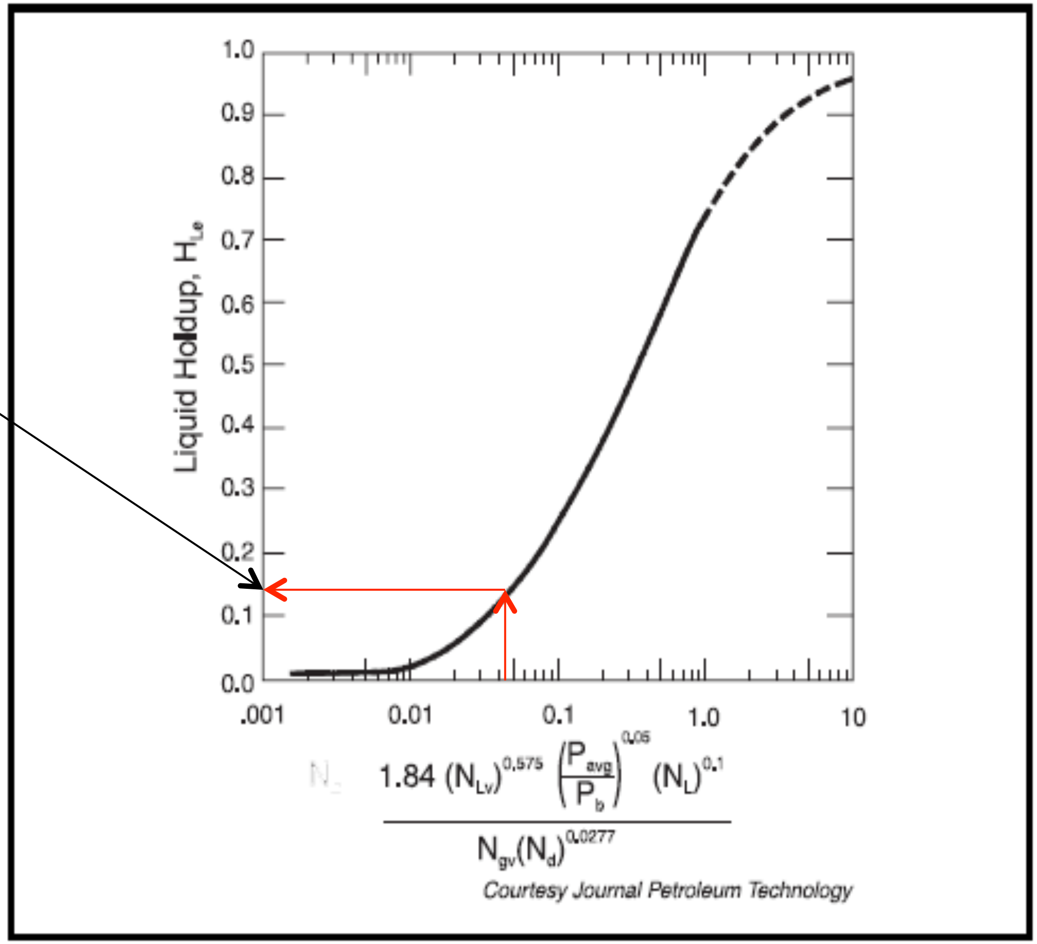
From chart:

Holdup fraction, $H_{Lo} = 0.14$.

Hold up – Eaton Method

0.14 or 14% of pipeline will contain liquids.

Eaton Liquid Holdup Correlation[21]



Worked Example – Pressure Drop Lockhart Martinelli

Pipeline in flowing with the following conditions

d_i = 15 ins = 0.381 m

Q_g = 29.5 ft^3/s = 0.835 m^3/s

Q_l = 1.3 ft^3/s = 0.0368 m^3/s

ρ_g = 8.39 lb/ft^3 = 134.4 kg/m^3

ρ_l = 35.57 lb/ft^3 = 569.8 kg/m^3

μ_g = 0.017 cp = 0.000017 Pa.s.

μ_l = 0.833 cp = 0.000833 Pa.s.

σ = 5 dyne/cm = 0.005 N/m.

Calculate pipe C.S.A. = 0.1139 m^2

∴ Superficial gas velocity, v_{sg} = 7.33 m/s

∴ liquid " , v_{sl} = 0.323 m/s

∴ Mixture velocity, $v_m = v_{sg} + v_{sl}$ = 7.65 m/s

Worked Example – Pressure Drop Lockhart Martinelli

LOCKHART MARTINELLI PRESSURE DROP. – 150 M pipe

Pipeline conditions as previous example.

Liquid Reynolds Number $N_{RE_L} = \frac{\rho v d}{\mu}$ = 84000

Gas Reynolds Number $N_{RE_g} = \frac{\rho v d}{\mu} = 2.20 \times 10^6$

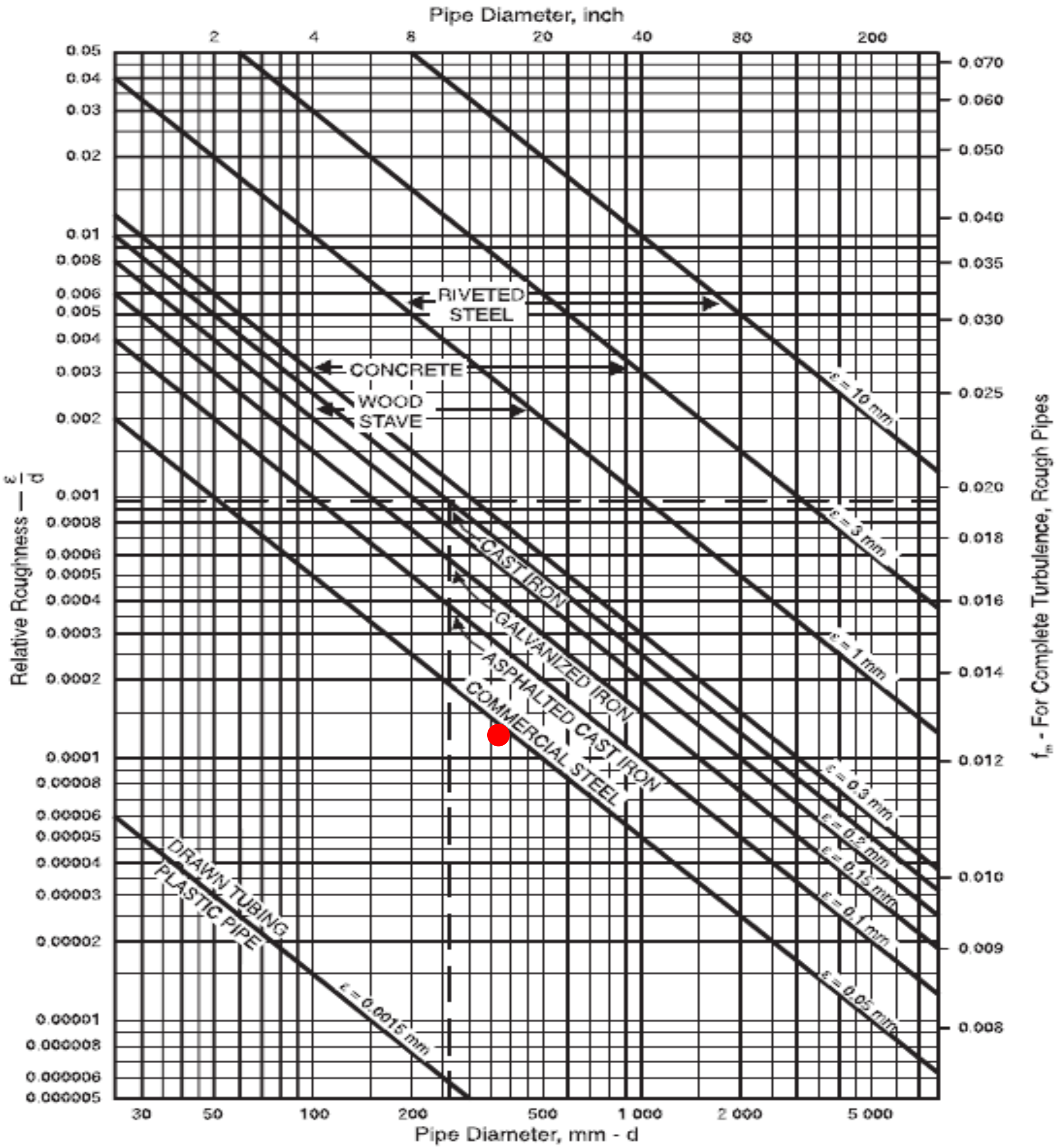
From roughness chart, relative roughness = 0.00012 (Commercial steel)

From Moody chart, friction factor liquid = 0.021

" " gas = 0.013

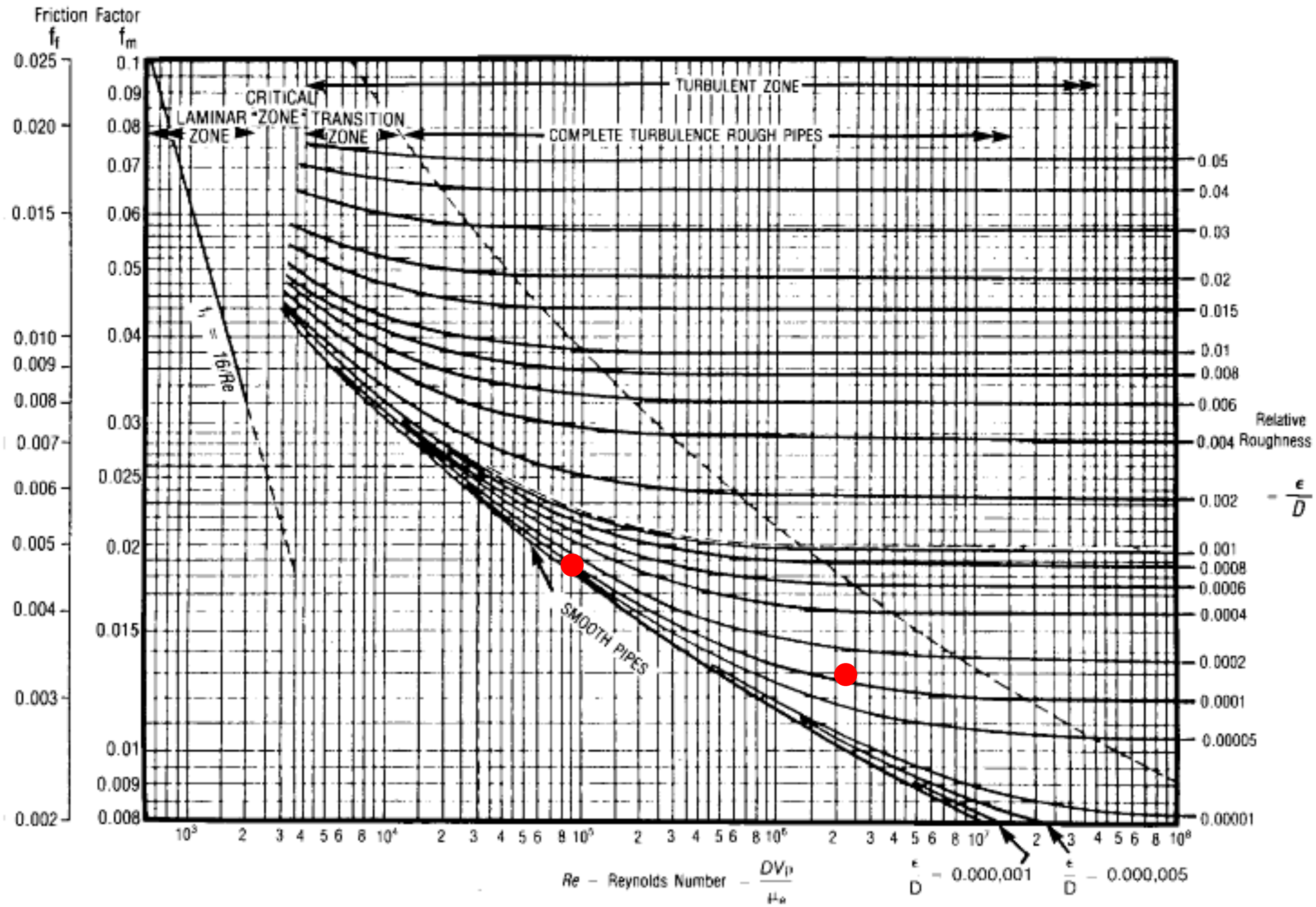
Liquid pressure drop, $\Delta P_L = f_l \frac{L \cdot \rho_L \cdot V_{SL}^2}{2d}$ = 1,642 Pa

Gas " , $\Delta P_g = f_g \frac{L \cdot \rho_g \cdot V_{Sg}^2}{2d}$ = 123.5 Pa



Pipe Diameter, inch
2
4
8
20
40
80
200
Relative Roughness — ε/d
0.05
0.04
0.03
0.02
0.01
0.008
0.006
0.005
0.004
0.003
0.002
0.001
0.0008
0.0006
0.0005
0.0004
0.0003
0.0002
0.0001
0.00008
0.00006
0.00005
0.00004
0.00003
0.00002
0.00001
0.000008
0.000006
0.000005
RIVETED STEEL
CONCRETE
WOOD STAVE
CAST IRON
GALVANIZED IRON
ASPHALTED CAST IRON
COMMERCIAL STEEL
DRAWN TUBING
PLASTIC PIPE
ε = 10 mm
ε = 3 mm
ε = 1 mm
ε = 0.3 mm
ε = 0.2 mm
ε = 0.15 mm
ε = 0.1 mm
ε = 0.05 mm
ε = 0.0015 mm
0.070
0.060
0.050
0.040
0.035
0.030
0.025
0.020
0.018
0.016
0.014
0.012
0.010
0.009
0.008
f_m - For Complete Turbulence, Rough Pipes
30
50
100
200
500
1 000
2 000
5 000
Pipe Diameter, mm - d

Worked Example – Pressure Drop Lockhart Martinelli



Worked Example – Pressure Drop Lockhart Martinelli

L-M parameter $X = \left[\dfrac{\left(\frac{dP}{dL}\right)_L}{\left(\frac{dP}{dL}\right)_g}\right]^{1/2} = 0.115$

$$Y_L = 1 + \frac{20}{X} + \frac{1}{X^2} = 249.3$$

$$\left(\frac{\Delta P}{L}\right)_{TP} = Y_L \cdot \left(\frac{\Delta P}{L}\right)_L = 249.3 \times 1.642 = 410 \text{ pa/m}$$

$$\Delta P = 410 \times 150 = 61500 \text{ pa} = 0.615 \text{ bar.}$$

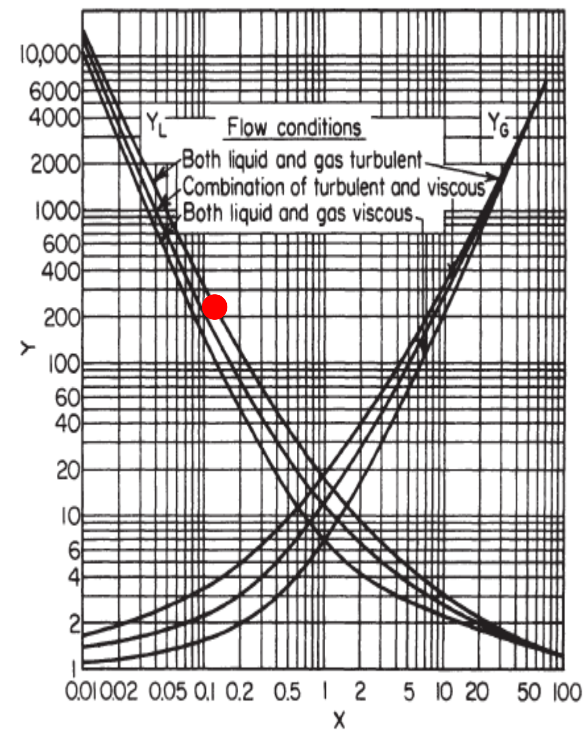


FIG. 6-26 Parameters for pressure drop in liquid/gas flow through horizontal pipes. (*Based on Lockhart and Martinelli,* Chem. Engr. Prog., **45**, 39 [*1949*].)

Two-Phase Flow - Beggs and Brill

The correlation presents different values for liquid holdup for each horizontal flow regime. Initially, the liquid holdup is calculated assuming the pipe is horizontal and then the value is corrected for the actual pipe inclination.

The correlation is modified to include transition zone between segregated and intermittent flow regimes.

A two-phase friction factor, independent of flow regime but a function of holdup is calculated.

Flow Regime Determination

The following variables are used to determine which flow regime would exist if the pipe were horizontal. This flow regime is a correlating parameter and does not reflect the actual flow regime.

Froude Number: $N_{fr} = V_m^2 / (g.d)$

Liquid Velocity Number $N_{Lv} = v_{sl} \,.\, (\rho_l \,.\sigma)^{0.25}$

$\lambda_l = v_{sl} / v_m$ and $\lambda_g = v_{sg} / v_m$

Two-Phase Flow - Beggs and Brill

Flow regime determination

$$L_i = p_i \lambda_l^{q_i}$$

i	p_i	q_i
1	316	0.302
2	0.0009252	-2.4684
3	0.10	-1.4516
4	0.5	-6.738

L_1, L_2, L_3 and L_4 are calculated and the horizontal flow regime is identified from the following limits;

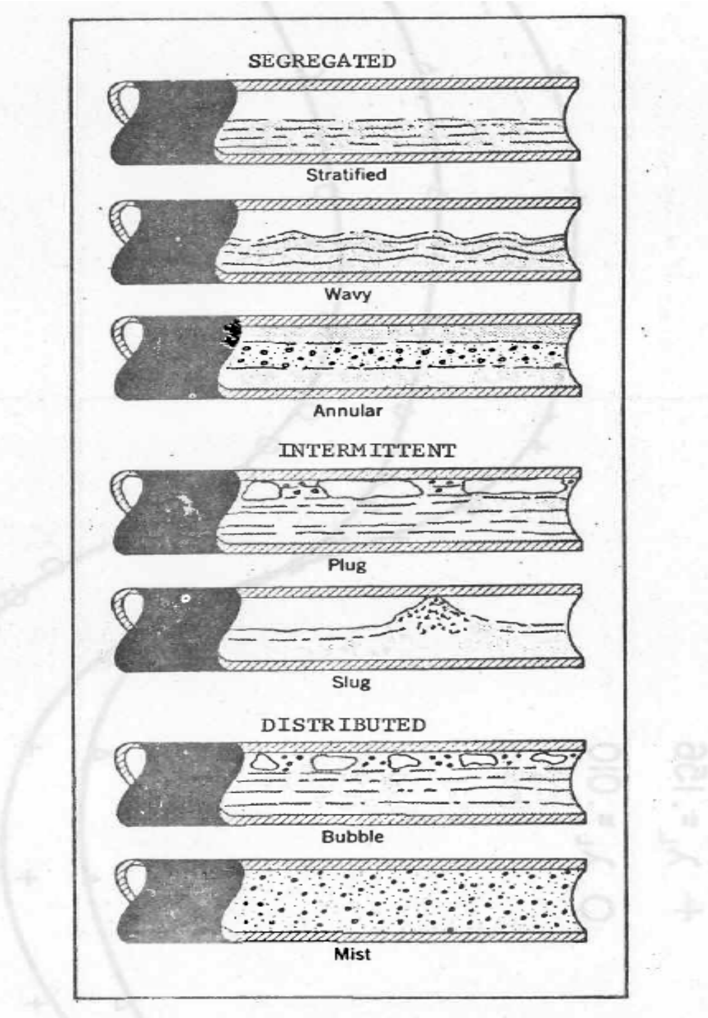
Segregated
$\lambda_l < 0.01$ and $N_{fr} < L_1$ or $\lambda_l \geq. 0.01$ and $N_{fr} < L_2$

Transition
$\lambda_l \geq 0.01$ and $L_2 \leq N_{fr} \leq L_3$

Intermittent
$0.01 \leq \lambda_l < 0.4$ and $L_3 < N_{fr} \leq L_1$ or $\lambda_l \geq 0.4$ and $L_3 < N_{fr} \leq L_4$

Distributed
$\lambda_l < 0.4$ and $N_{fr} \geq L_1$ or $\lambda_l \geq 0.4$ and $N_{fr} > L_4$

Two-Phase Flow - Beggs and Brill Flow Regimes



When the flow falls in the transition regime the liquid holdup must be calculated using both the segregated and intermittent and interpolated with weighting factors.

Two-Phase Flow - Beggs and Brill

Two-phase Density:
For a pipe at an angle θ from horizontal the holdup in the inclined pipe is a function of the holdup in the pipe if horizontal:

$$H_{l(\theta)} = H_{l(0)}\ \psi$$

$$H_{l(0)} = a\ \lambda_l^{\,b}/N_{fr}^{\,c}$$

Flow Pattern	a	b	c
Segregated	0.98	0.4846	0.0868
Intermittent	0.845	0.5351	0.0173
Distributed	1.065	0.5824	0.0609

Example: For segregated flow regime

$$H_{l(0)} = 0.98\ \lambda_l^{\,0.4846}/N_{fr}^{\,0.0868}$$

Two-Phase Flow - Beggs and Brill

$$\psi = 1 + C\,[\sin(1.8\,\theta) - 0.333\,\sin^3(1.8\,\theta)]$$

For vertical upward flow, $\theta = 90$ and hence:

$$\psi = 1 + 0.3\,C$$

Where C is defined as:

$$C = (1 - \lambda_l)\ln(d\,\lambda_l^{e}\,N_{lv}^{f}\,N_{fr}^{g})$$

Where d, e, f and g are specified for each flow regime as:

Horizontal Flow Pattern	d	e	f	g
Segregated Uphill	0.011	-3.768	3.539	-1.614
Intermittent Uphill	2.96	0.305	-0.4473	0.0978
Distributed Uphill	No Correction $C=0$ $\psi=1$, $h_l \neq f(\theta)$			
All flow patterns Downhill	4.70	-0.3692	0.1244	-0.5056

Two-Phase Flow - Beggs and Brill

Friction Presure Gradient

$$(dp/dZ)_f = f_{tp}\, \rho_n\, v_m^2 / (2\, g_c\, d)$$

where

$$\rho_n = \rho_l\, \lambda_l + \rho_g\, \lambda_g$$

$$f_{tp} = f_n\, (f_{tp}/f_n)$$

The non-slip friction factor is determined from the smooth pipe curve on a Moody diagram or from:

$$f_n = 1 / [\, 2 \log(N_{Ren}/(4.5223 \log N_{Ren} - 3.8215))]^2$$

Using the Reynolds number:

$$N_{Ren} = \rho_n\, v_m\, d / \mu_n \quad \text{(volume average no slip viscosity and density)}$$

The ratio of the two-phase to the non-slip friction factor is calculated by

$$f_{tp}/f_n = \exp(s)$$

where

$$s = \ln(2.2.\, y - 1.2)$$

and

$$y = \lambda_l/[H_{L(\theta)}]^2$$

Beggs and Brill – Worked Example

Example problem by Beggs - Brill Method.

Given the following information for a wet gas pipeline, calculate the pressure gradient due to friction. Flow is horizontal.

$\mu_o = 1.359$ cp $\quad \mu_g = 0.0233$ cp.

$\sigma_o = 4.608$ dynes/cm.

$\rho_o = 42.45$ lb/ft^3 $\quad \rho_g = 13.66$ lb/ft^3

$d = 16$ ins $= 1.333$ ft.

$P = 2500$ psia.

$T = 60°F$

$V_{SL} = 0.264$ ft/s $\quad V_{sg} = 12.867$ ft/s.

1. Determine flow regime.

$N_{FR} = V_m^2/gd \qquad V_m = V_{SL} + V_{sg} = 13.131$ ft/s.

$= (13.131)^2/(32.2 \times 1.33)$

$= 4.02.$

$\lambda_L = \dfrac{V_{SL}}{V_{SM}} = \dfrac{0.264}{13.131} = 0.020$

Beggs and Brill – Worked Example

$$L_1 = 316\,\lambda_L^{0.302} = 96.96$$

$$L_2 = 0.0009252\,\lambda_L^{-2.4684} = 14.45$$

$$L_3 = 0.10\,\lambda_L^{-1.4516} = 29.25$$

$$L_4 = 0.50\,\lambda_L^{-6.738} = 25.6$$

$$\left.\begin{array}{ll} \lambda_L \geq 0.01 & \text{yes} \\ N_{FR} < L_2 & \text{yes} \end{array}\right\} \quad \underline{\text{flow is segregated.}}$$

2. Determine Liquid Hold up.

$$H_{L(0)} = 0.98 \times \frac{\lambda_L^{0.4846}}{N_{FR}^{0.0868}}$$

$$= 0.98 \times \frac{0.15}{1.128}$$

$$= 0.13$$

3. Determine two-phase friction factor

average no slip density, $\rho_n = 0.02 \times 42.45 + (1-0.02) \times 13.66$

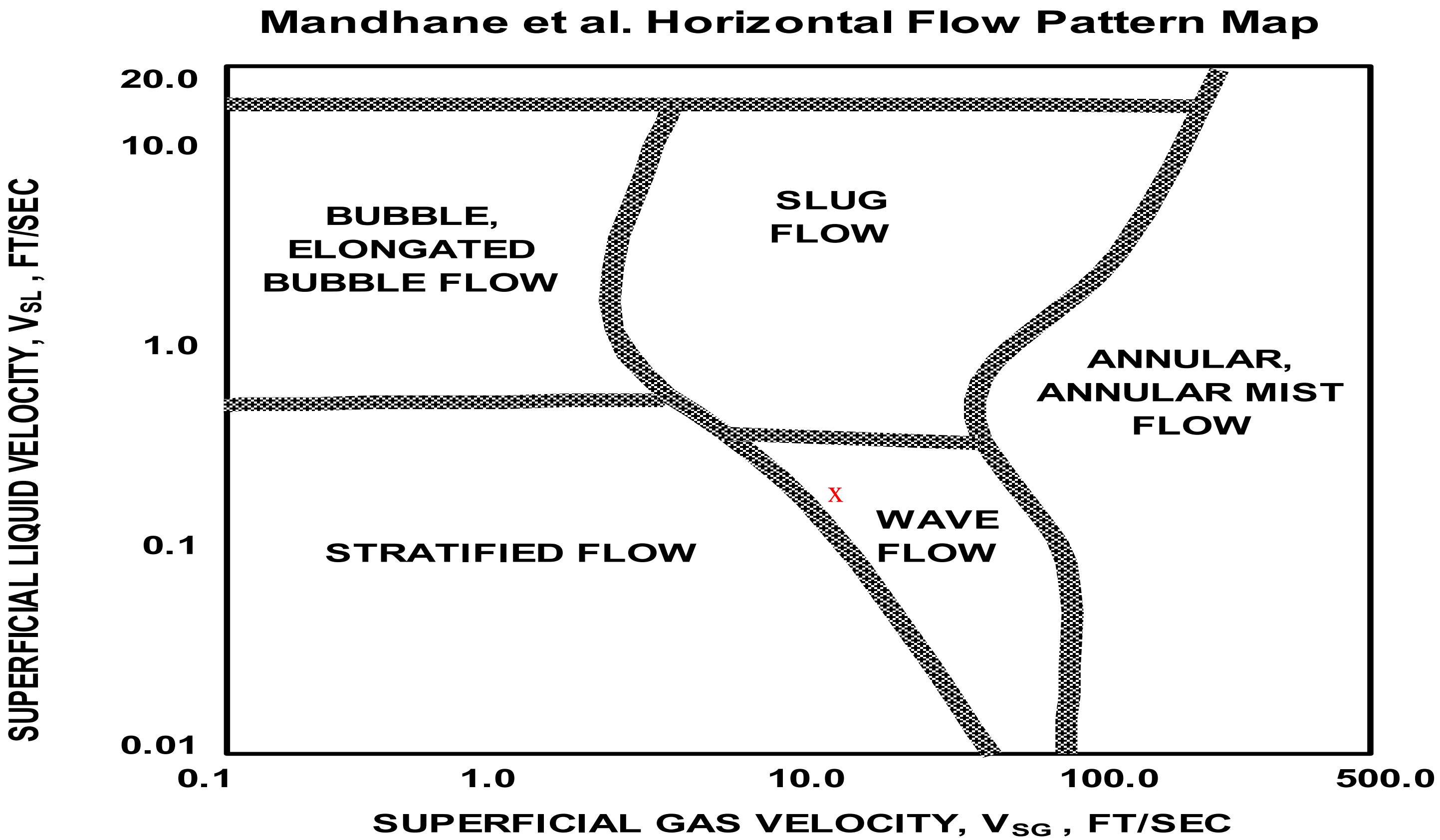
$= 14.236\ \text{lb/ft}^3$

.. viscosity, $\mu_n = 0.02 \times 1.4 + (1-0.02) \times 0.0233$

$= 0.05\ \text{cp}$

Beggs and Brill – Worked Example

Aligns with Mandhane map.



Beggs and Brill – Worked Example

$$N_{Rem} = 1488 \cdot \frac{\ell_n v_m d}{\mu_n} \quad (1488 \text{ unit correction})$$

$$= \frac{1488 \times 14.236 \times 13.131 \times 1.333}{0.05}$$

$$= 7.416 \times 10^6$$

$$f_n = 1 / \left[2 . \log\left(N_{Rem} / (4.5223 . \log N_{Rem} - 3.8215)\right)\right]^2$$

$$= 1 / \left[2 \log\left(7.416 \times 10^6 / 27.25\right)\right]^2$$

$$= 0.00846$$

$$y = \lambda_L / (H_{L(0)})^2 = \frac{0.02}{0.13^2} = 1.183$$

$$S = \ln(2.2 y - 1.2)$$

$$= 0.338$$

$$\frac{f_{tp}}{f_n} = e^S = 1.402$$

$$f_{tp} = 1.402 \times 0.00846 = 0.01187$$

Beggs and Brill – Worked Example

4. Determine frictional pressure drop

$$\left(\frac{dP}{dZ}\right)_f = \frac{f_{tp} \cdot \rho_m \cdot V_m^2}{2 \cdot g \cdot d}$$

$$= \frac{0.01187 \times 14.236 \times (13.131)^2}{2 \times 32.2 \times 1.333}$$

$$= 0.34 \text{ psf/ft}$$

$$= 0.00236 \text{ psi/ft}$$