1 Heat Exchangers

1.1 Introduction

A heat exchanger can be defined as any device that facilitates the transfer of thermal energy between two or more fluids at different temperatures. These fluids may be allowed to mix directly, or as is more commonly the case, they may be separated by some physical barrier such as a pipe wall. They can be found in all manner of industrial applications from air-conditioning to electronics to chemical processing plants and range greatly in size and complexity. Take, for example, the laptop that was used to write these notes. It contains a (relatively) small fan which acts to exhaust hot air from the casing in order to prevent the central processing unit from overheating. At the other end of the scale, you may find heat exchangers which are several meters long, weighing thousands of kilograms, in many industrial processing plants.

In the following sections we will start by taking a more in-depth look at the different ways we can ***classify heat exchangers***. We will then discuss one of the main characteristics of a heat exchanger, the ***overall heat transfer coefficient*** before moving on to look at two common approaches used in the analysis of heat exchangers, namely the ***log mean temperature difference method*** and the ***effectiveness-NTU method***.

1.2 Classification of heat exchangers

At the broadest level, heat exchangers can be separated into two main types:

1. recuperative and;
2. regenerative.

With ***recuperative*** devices, heat transfer occurs between different fluid streams through a separating surface such as a pipe wall (or a fluid-fluid interface if the fluids are immiscible) and ideally the fluid streams do not mix or leak. Examples of recuperative heat exchangers include the double-pipe (concentric tube) and shell-and-tube type, which are described in greater detail later in this section. With ***regenerative***, or storage, type devices on the other hand, the same flow chamber (matrix) is alternately occupied by one of the two fluids. Thus during the first phase of the cycle, when the hot fluid is occupying the flow chamber, it acts to heat the surroundings. Then during the next phase, when the cold fluid replaces the hot fluid inside the chamber, it extracts some of that thermal energy that was built-up during the previous phase. The process then repeats periodically. Rotary and fixed-matrix type models are examples of regenerative heat exchangers. In this introductory module we will focus only on recuperative heat exchangers, since they constitute the vast majority of heat exchangers encountered in practice.

At a more detailed level, heat exchangers are also commonly classified according to the following main criteria:

- number of fluids (e.g. 2-fluid, 3-fluid, n-fluid);
- transfer processes (e.g. direct versus indirect contact types);
- heat transfer mechanisms (e.g. combined convective & radiative heat transfer);
- flow arrangements (e.g. single pass, multi-pass, cross-flow, parallel flow) and;
- construction (e.g. tubular, plate-type, extended surface).

Appendix A provides some hierarchical illustrations of the different types of heat exchangers that are grouped into these five categories. The reader is also directed to the textbooks of Shah and Selukić (2003) and Kakaç *et al.* (2012) for a more extensive description of these different heat exchanger classifications.

In closing the section, we return to discuss two common heat exchanger configurations, that of the double-pipe (concentric tube) and shell-and-tube type heat exchanger. Schematics of these heat exchangers are shown in Figure 1 and 2.

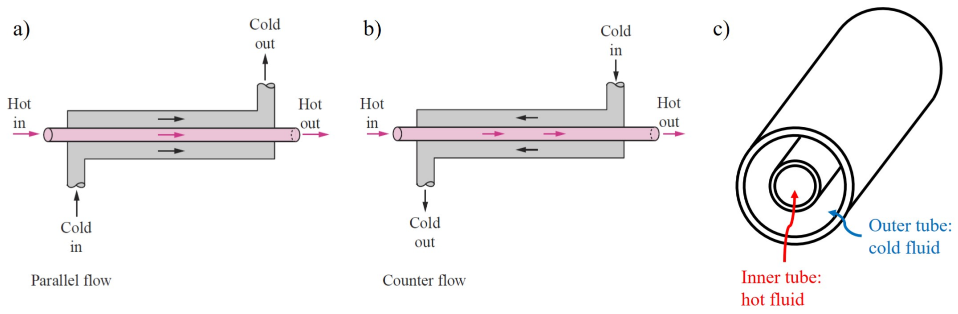


Figure 1: A double-pipe (concentric tube) heat exchanger in a) a parallel flow configuration and b) a counter-flow configuration. c) A three-dimensional cross sectional view of the concentric tube arrangement (adapted from Çengel, 2002)

Looking firstly at the ***double-pipe heat exchanger***, we see that it consists of two concentric pipes of different diameter, with one fluid travelling through the inner pipe while a separate fluid is transported through the space between the inner and outer pipes. When the fluids are at different temperatures, thermal energy is exchanged across the wall of the inner pipe. When the fluid streams flow in the same direction between the inlet and the outlet, the heat exchanger is said to be operating in a *parallel* flow configuration. Conversely, in a *counter-flow* configuration, the fluid streams travel in opposing directions through the heat exchanger.

Looking next at the typical ***shell-and-tube heat exchanger*** (Figure 2a), we see that it consists of an outer shell which houses a number of tubes (sometimes several hundred of them). One of our fluids travels through the tubes while the other flows through the shell, external to the tubes, and in doing so heat is exchanged across the tube walls. Sometimes baffles may also be installed within the shell to encourage the shell-side fluid to flow fully across the shell and thus maximise the heat transfer. A further distinction can be made with these heat exchangers depending on the number of passes that the fluids make while travelling between their respective inlets and outlets. For example, the diagram in Figure 2b shows a '*one-shell and two-tube*' pass heat exchanger. Other multiples of this configuration may also be encountered e.g. '*two-shell and four-tube*' (Figure 2c) and so on.

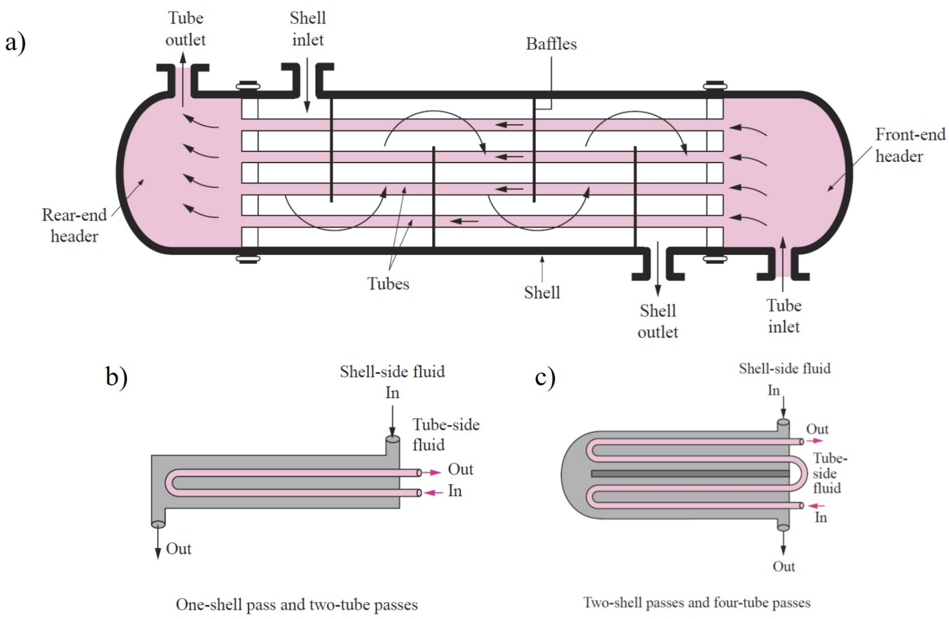


Figure 2: a) Schematic of a typical shell-and-tube heat exchanger. b) Example of a 'one-shell and two-tube pass' heat exchanger. c) Example of a 'two-shell and four-tube pass' heat exchanger (adapted from Çengel, 2002)

1.3 Overall heat transfer coefficient

We have started to develop a sense of the ubiquity and variety of heat exchanger technologies at our disposal. We may then reasonably ask how we can distinguish the suitability of one particular heat exchanger from another. There are many ways to do this and each situation will merit its own unique set of considerations. However, in general, one of the most useful measures we can refer to is the overall heat transfer coefficient, U. The parameter U appears in the general expression for heat flux

$$Q = UA\Delta T \tag{1}$$

Equation (1) shows that the amount of heat transfer is dependent on the available temperature gradient ΔT, as well as the available area A over which the transfer takes place and lastly, the overall heat transfer coefficient U. We can also note that U is the reciprocal of R, which we use to represent the combined thermal resistance of our system,

$$R = \frac{1}{UA} \tag{2}$$

You may have already encountered the idea of combined thermal resistance (e.g. heat transfer through an insulated brick wall). In the context of heat exchangers, perhaps a couple of more relevant examples for our considerations is that of two fluids at different temperatures: (1) separated by a plane wall (plate) or; (2) flowing in a double-pipe (concentric tube) arrangement. These scenarios are depicted in Figure 3.

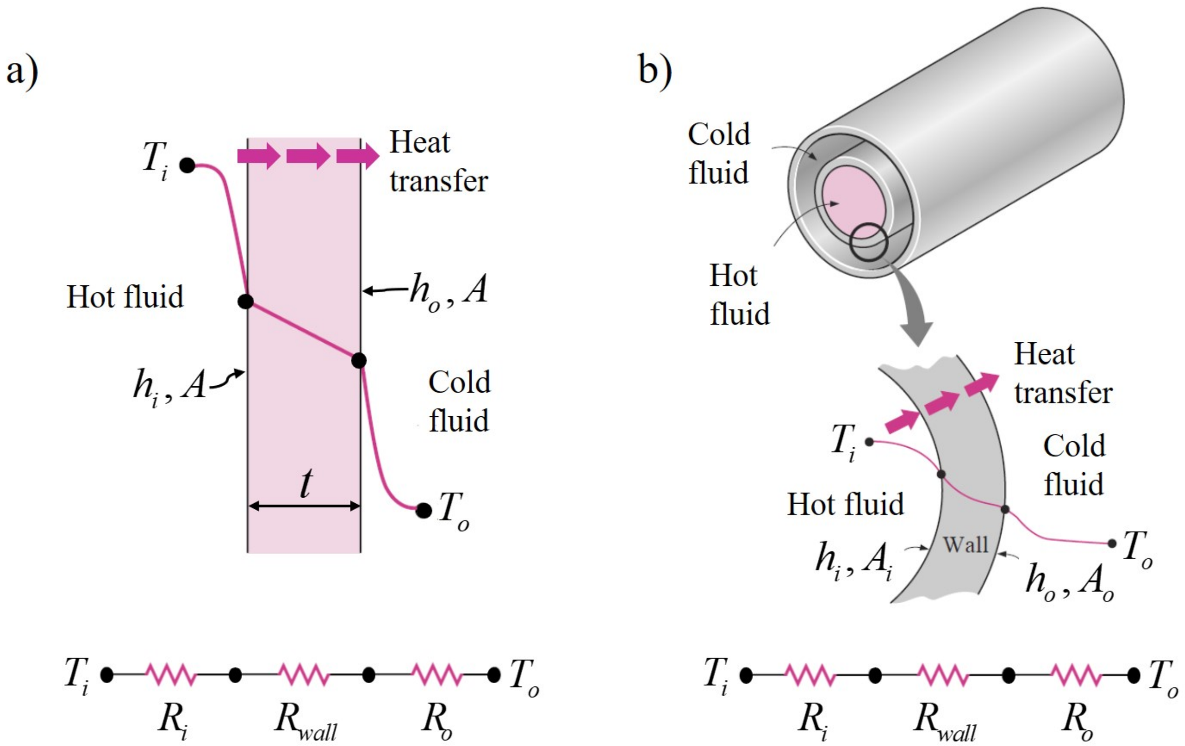


Figure 3: Heat transfer and corresponding thermal resistance across a) a plane wall (plate) and b) a double-pipe (concentric tube) arrangement. In both cases the combined thermal resistance is seen to consist of an inner (R_i), an outer (R_o) and a wall (R_{wall}) component (adapted from Çengel, 2002)

In both cases, the total thermal resistance, R from the outer to the inner fluid is,

$$R = R_i + R_{wall} + R_o. \tag{3}$$

For the case of the plane wall (plate), of thickness t and thermal conductivity k, separating the fluids, we have

$$R_i = \frac{1}{h_i A};\ R_{wall} = \frac{t}{kA};\ R_o = \frac{1}{h_o A} \tag{4}$$

where h_i and h_o are the heat transfer coefficients of the inner and outer fluids, respectively. While for the concentric tube of length L the thermal resistances are

$$R_i = \frac{1}{h_i A_i};\ R_{wall} = \frac{\ln\left(D_o/D_i\right)}{2\pi L k};\ R_o = \frac{1}{h_o A_o} \tag{5}$$

where $A_i = \pi D_i L$ and $A_o = \pi D_o L$, are the internal and outer superficial areas of the interior pipe, respectively.

Returning now to our expression for total heat flux, for the concentric tube heat exchanger we can write

$$Q = \frac{\Delta T}{R} = U_i A_i \Delta T = U_o A_o \Delta T \tag{6}$$

Eliminating ΔT and inverting Q yields,

$$\frac{1}{U_i A_i} = \frac{1}{U_o A_o} = R = [R_i + R_{wall} + R_o] \tag{7}$$

It is important to note that we have two values for our overall heat transfer coefficient, U_i and/or U_o, depending on which surface area we specify in our calculations e.g.

$$\frac{1}{U_i A_i} = \left[\frac{1}{h_i A_i} + \frac{\ln\left(D_o/D_i\right)}{2\pi L k} + \frac{1}{h_o A_o}\right] \tag{8}$$

and/or

$$\frac{1}{U_o A_o} = \left[\frac{1}{h_i A_i} + \frac{\ln\left(D_o/D_i\right)}{2\pi L k} + \frac{1}{h_o A_o}\right] \tag{9}$$

For the case of the hot and cold fluids separated by the plane wall (plate), we have $A_i = A_o = A$ and hence $U_i = U_o = U$ and the expression for U simplifies to:

$$\frac{1}{U} = \left[\frac{1}{h_i} + \frac{t}{k} + \frac{1}{h_o}\right] \tag{10}$$

Further simplifications can be made for '***thin walls***'. That is to say, if the thickness of our plane wall or pipe is small (in comparison with the length of the heat exchanger), with a large thermal conductivity, then the thermal wall resistance is negligible (i.e. $R_{wall} \sim 0$).

Further, this means we can assume $A_i \sim A_o \sim A$ (i.e. inner/outer diameters are very similar), and our expression for U reduces to:

$$\frac{1}{U} \sim \frac{1}{h_i} + \frac{1}{h_o} \tag{11}$$

All of the foregoing discussion is contingent upon the surface transfer areas of our heat exchanger being clean and free of debris. However, in reality during the lifetime of our heat exchanger, films of dirt or scale may build up on the heat-transfer surfaces as a result of various mechanisms as depicted in Figure 4.

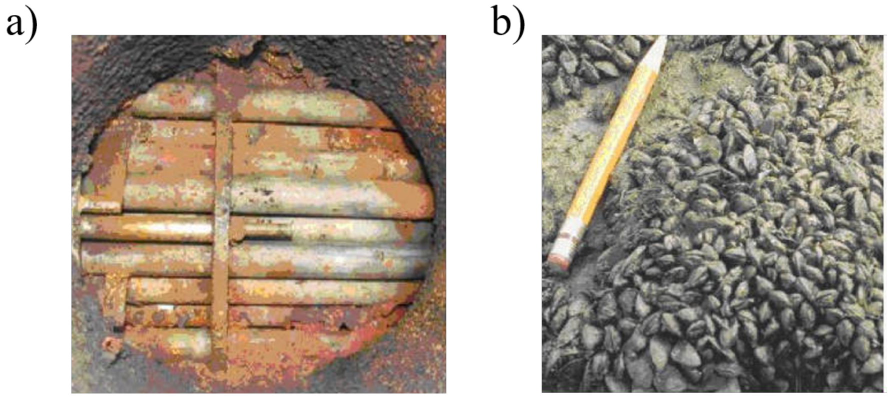


Figure 4: Examples of a) corrosion-based and b) biological-based fouling that can occur in heat exchangers

This is typically referred to as ***Fouling***, and it has a direct (negative) impact on the performance of our heat exchanger with time. To account for this in our analysis of the heat exchanger, we can introduce a fouling factor R_f as follows,

$$\frac{1}{U_i A_i} = \frac{1}{U_o A_o} = R = \frac{1}{h_i A_i} + \frac{R_{f,i}}{A_i} + R_{wall} + \frac{R_{f,o}}{A_o} + \frac{1}{h_o A_o} \tag{12}$$

In equation (12), we have specified separate fouling factors for the inner and outer heat transfer surfaces, respectively. The equation also shows that the more significant the degree of fouling, the greater the value of R_f will be, which in turn directly increases the overall thermal resistance of the system and hence decreases the heat transfer.

It is mentioned here in passing that upon examination of equation (2), an additional strategy can be adopted in order to improve overall heat transfer, namely, by increasing the heat transfer surface area, A. In practice, this can be achieved by adding fins to our heat transfer surfaces. When either side of our heat transfer surface is finned, then the total area becomes,

$$A = A_{fin} + A_{unfinned} \tag{13}$$

where A_{fin} is the surface area of the fins and $A_{unfinned}$ is the remaining heat transfer surface area. Finned surfaces are beyond the scope of this module but the interested reader is encouraged to research this topic further and is referred to the suggested further reading section at the end of these notes for some source material to start with.

1.4 Analysis of heat exchangers

In the analysis that follows, we will make a number of simplifying assumptions which will greatly reduce the complexity of the problems that we are trying to solve. For example, since heat exchangers typically operate for long periods of time with no change in their operating conditions, we can consider them to be steady state. This in turn means that we can consider mass flow rate of each fluid in our heat exchanger to be constant. Furthermore, fluid properties such as inlet and outlet temperatures and velocities, as well as thermo-physical properties such as heat capacity, all remain the same throughout while changes to the kinetic and potential energy of the fluid flows are negligible. Lastly, we regard the outer surface of the heat exchanger to be fully insulated such that all heat transfer occurs only between the fluids within the confines of the heat exchanger. For most situations, these assumptions are reasonable but we should always keep them in mind when interpreting the accuracy or validity of our results.

1.4.1 Log mean temperature difference method

So far, we have seen the basic working principles of heat exchangers and we have carried out some calculations to estimate their overall heat transfer coefficient, U , which we can relate to the total heat flux through $Q = UA\Delta T$. However, we have not yet touched upon the driving mechanism of the heat transfer process, that is the temperature gradient, ΔT.

Our starting point is to recognise that in all but a few exceptional situations, the temperature difference between the fluids in our heat exchanger will actually change along its length. Therefore, it is perhaps more appropriate to consider a suitable mean temperature difference, ΔT_m, when trying to calculate the total heat flux Q.

In order to develop this idea we will start with the simplest cases of parallel and counter-flow heat exchangers, where the thermal energy transfer is taking place between a hot fluid and a cold fluid. Diagrams of the temperature change of the fluids between the inlet and outlet of the heat exchangers are shown in Figure 5.

For the case of the parallel flow heat exchanger, we see that the hot and cold fluid streams arrive at the inlet with temperatures $T_{h,in}$ and $T_{c,in}$, respectively, and the resulting

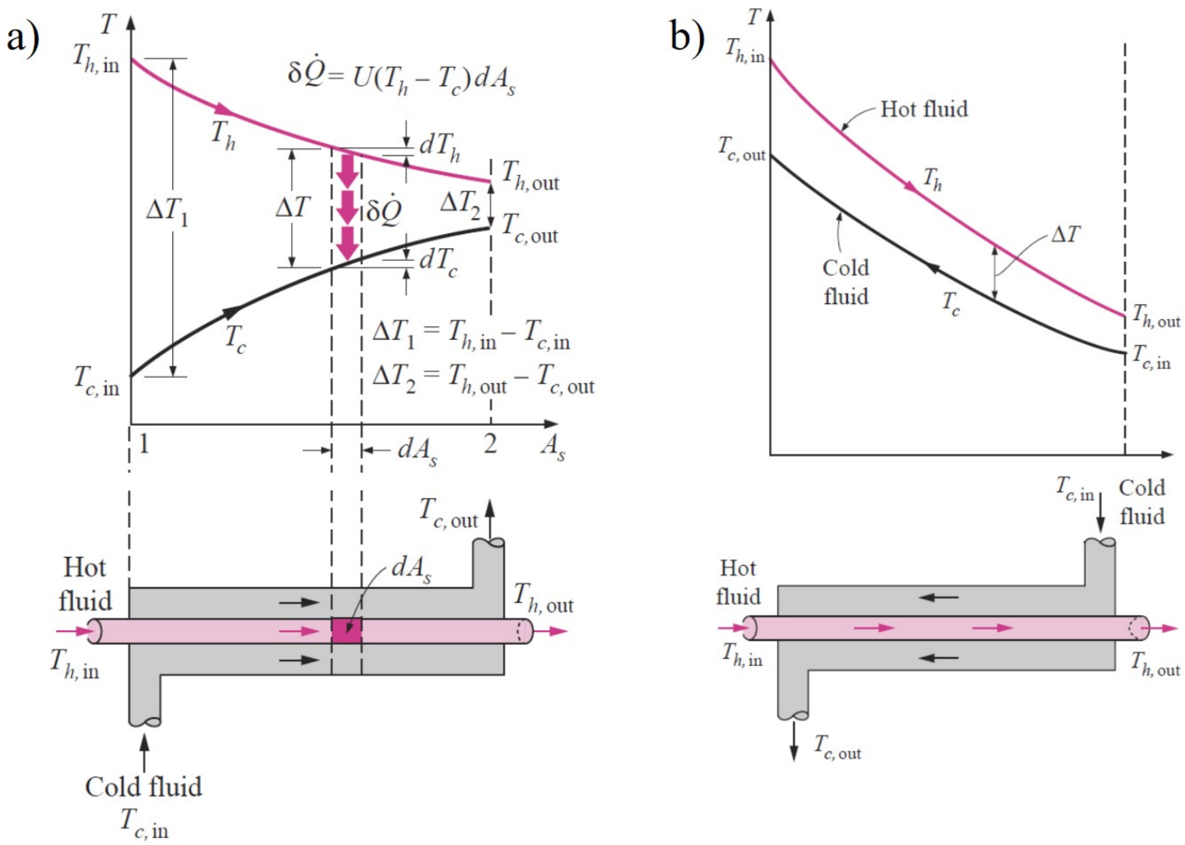


Figure 5: Temperature change of cold and hot fluid streams between the inlets and outlets of a) a parallel flow and b) a counter-flow heat exchanger (adapted from Çengel, 2002)

temperature difference between them, ΔT_1, is at its maximum. Then, as the fluids pass through the heat exchanger, the cold fluid stream temperature increases at the expense of the hot fluid stream temperature and by the time the fluids have reached the outlet, the temperature difference, ΔT_2 has reduced considerably relative to ΔT_1. In fact, the local temperature difference decreases exponentially along the heat exchanger. By considering an energy balance to a differential element dA_s at a point somewhere along the heat exchanger, it is possible to derive the appropriate expression for ΔT_m, which turns out to be of the form,

$$Q = UA\Delta T_m \tag{14}$$

where,

$$\Delta T_m = \left[\frac{\Delta T_1 - \Delta T_2}{\ln\left(\Delta T_1/\Delta T_2\right)}\right] \tag{15}$$

and

$$\Delta T_1 = T_{h,in} - T_{c,in} \tag{16}$$

$$\Delta T_2 = T_{h,out} - T_{c,out} \tag{17}$$

Note that in equation (16) the subscripts h and c are used to denote our hot and cold fluid streams, respectively while the subscripts *in* and *out* refer to the inlet and outlet of the heat exchanger. The precise derivation of these expressions is omitted here for brevity but the interested reader can find full information in any relevant textbook (please refer to the section on further reading at the end of these notes) or reliable online source. A similar derivation can also be applied to the counter-flow heat exchanger with the end result varying only slightly, wherein the temperature gradient terms, ΔT_1 and ΔT_2 in equation (16) are given instead by,

$$\Delta T_1 = T_{h,in} - T_{c,out} \tag{18}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} \tag{19}$$

The LMTD method we have just outlined is only strictly applicable to parallel- and counter-flow heat exchangers, respectively. Due to the complexity of similar expressions for cross-flow and multi-pass shell-and-tube heat exchangers, a more convenient approach in the form of a correction factor, F, has been developed. According to this method we can relate the temperature difference to its equivalent LMTD relation for counter-flow heat exchangers, e.g.

$$\Delta T_m = F \Delta T_{m,CF} \tag{20}$$

We note that F depends on the geometry of the heat exchanger and the inlet/outlet temperatures of the hot and cold fluid streams. For cross-flow and multi-pass heat exchangers $F \leq 1$, with the limiting value of 1 corresponding to a counter-flow heat exchanger. So we can think of F as being a measure of the deviation of the log-mean temperature ΔT_m from the corresponding value for a counter-flow heat exchanger.

The value of F has been determined for a variety of standard heat exchanger configurations and these are available in the form of correction factor charts, an example of which is shown in Figure 6 (see also Appendix B). From Figure 6 we see that F is expressed as a function of two temperature ratios i.e. $F = f(P, R)$, where,

$$R = \frac{T_{in} - T_{out}}{t_{out} - t_{in}} = \frac{(\dot{m}C_p)_{\text{tube-side}}}{(\dot{m}C_p)_{\text{shell-side}}} \tag{21}$$

and

$$P = \frac{t_{out} - t_{in}}{T_{in} - t_{in}} \tag{22}$$

In equations (21) and (22) the variables T and t are used to denote the shell- and the tube-side temperatures, respectively, while in equation (21) $\dot{m}$ is the mass flow rate and C_p

is the specific heat of the respective shell- and tube-side fluids. Importantly, we see that the use of the correction factor requires us to know both the inlet and outlet temperatures of both our hot and cold fluids.

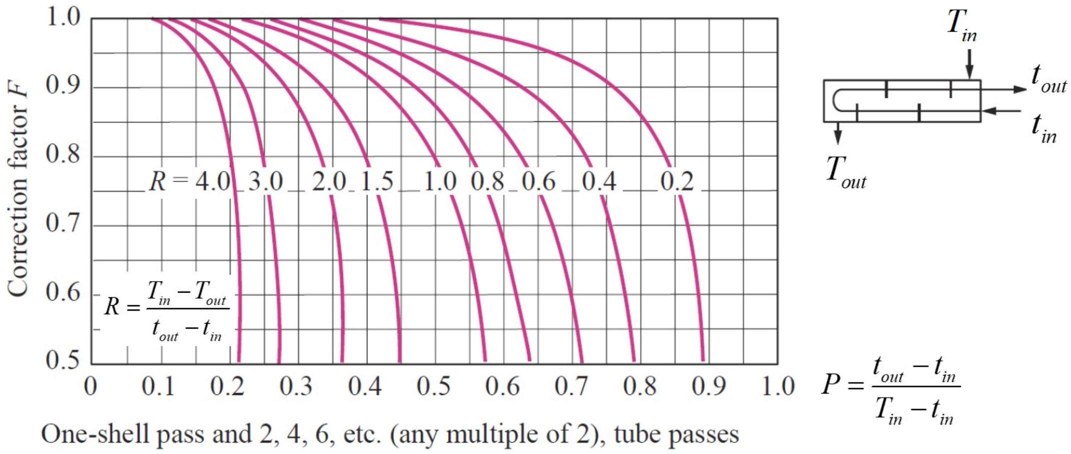


Figure 6: Example of a correction factor chart for a heat exchanger with one-shell pass (adapted from Çengel, 2002)

1.4.2 The effectiveness-NTU method

The LMTD method is straightforward to use when the inlet and outlet temperatures of the hot and cold fluids are known or can be determined from an energy balance. When this is not the case and we want to know if a specified heat exchanger can perform our required heat transfer rates then the LMTD method becomes iterative (and tedious). The effectiveness-NTU method was developed by Kays and London in 1955 to help solve such problems in an efficient manner.

The effectiveness-NTU method is based on a dimensionless parameter called the heat transfer effectiveness, ε, defined as

$$\varepsilon = \frac{Q}{Q_{\max}} = \frac{\text{actual heat transfer rate}}{\text{maximum possible transfer rate}} \tag{23}$$

The maximum possible heat transfer rate in our heat exchanger is

$$Q_{\max} = C_{\min}\left(T_{h,in} - T_{c,in}\right) \tag{24}$$

where $C_{\min} = \min\left\{\dot{m}_c C_{pc}, \dot{m}_h C_{ph}\right\}$. With knowledge of the inlet temperatures and mass flow rates of the hot and cold fluids, we can calculate the actual heat transfer rate Q

without knowing anything about the outlet temperatures e.g.

$$Q = \varepsilon Q_{\max} = \varepsilon C_{\min} \left(T_{h,in} - T_{c,in}\right) \tag{25}$$

Of course we still need to determine the effectiveness, ε.

The effectiveness of a heat exchanger depends on the geometry of the heat exchanger as well as the flow arrangement. Therefore, different types of heat exchangers have different effectiveness relations. Such effectiveness relations have been developed for a large number of heat exchangers, but they can get rather complicated (see Appendix D). Fortunately, the results are also available graphically (see Appendix C). An example of an effectiveness chart for a parallel and a counter-flow heat exchanger is shown in Figure 7. You will see that ε is given as a function of two dimensionless parameters, i.e. $\varepsilon = f(NTU, c)$ where NTU or Number of Transfer Units is

$$NTU = \frac{UA}{C_{\min}} = \frac{UA}{(\dot{m}C_p)_{\min}} \tag{26}$$

and c is the capacity ratio,

$$c = \frac{C_{\min}}{C_{\max}} \tag{27}$$

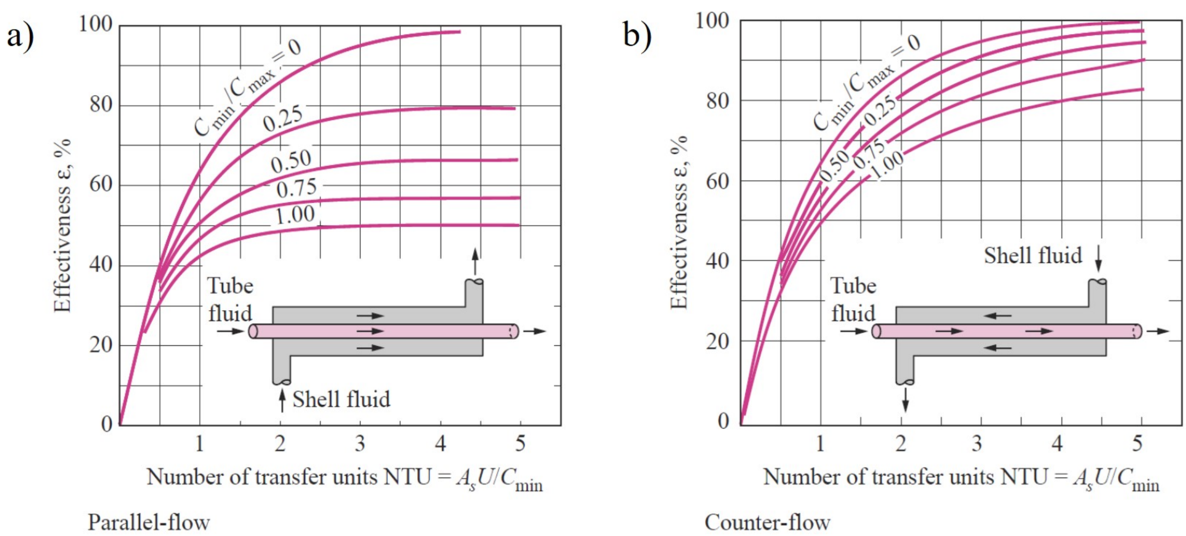
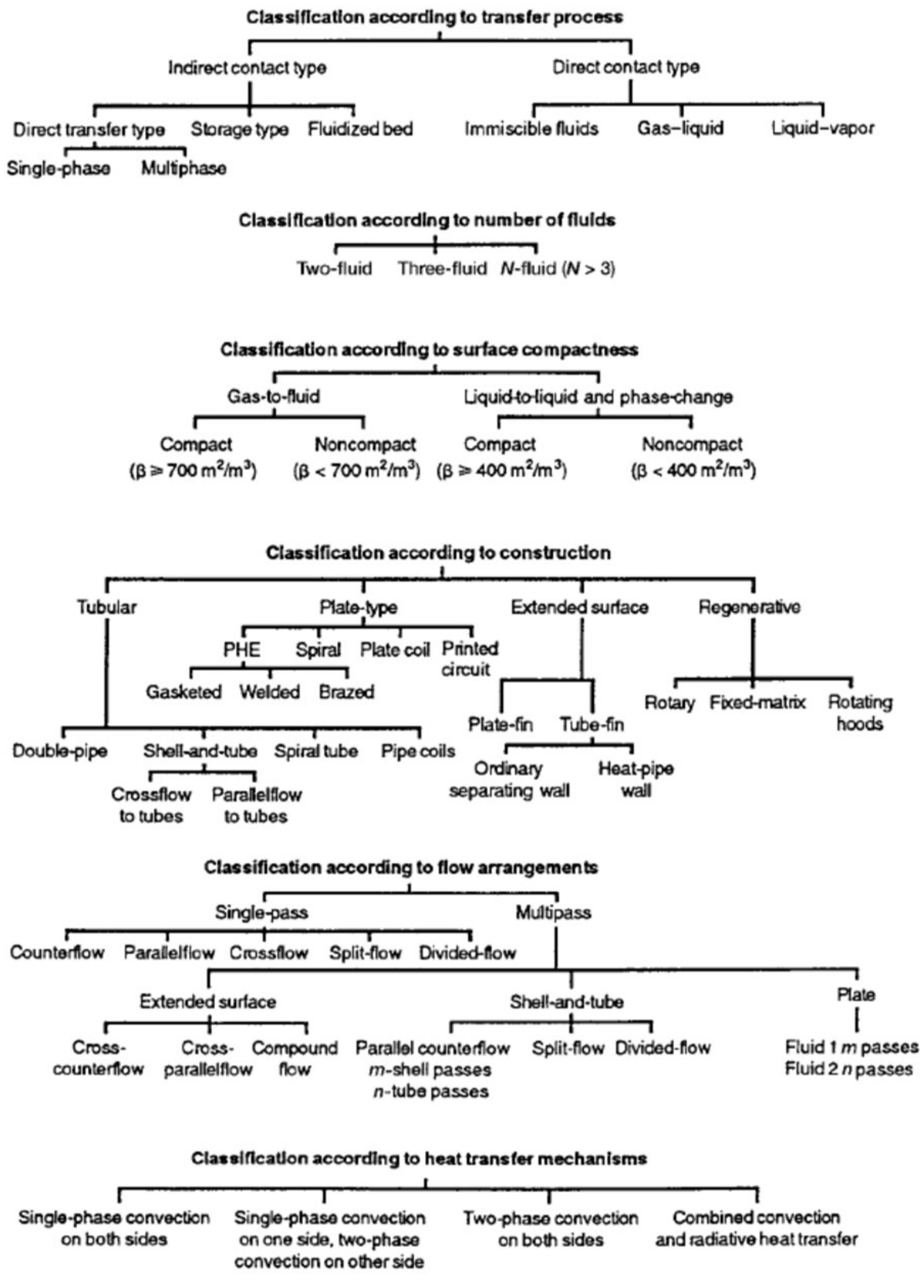


Figure 7: Effectiveness charts for a) parallel flow and b) counter-flow heat exchangers (adapted from Çengel, 2002)

Further reading

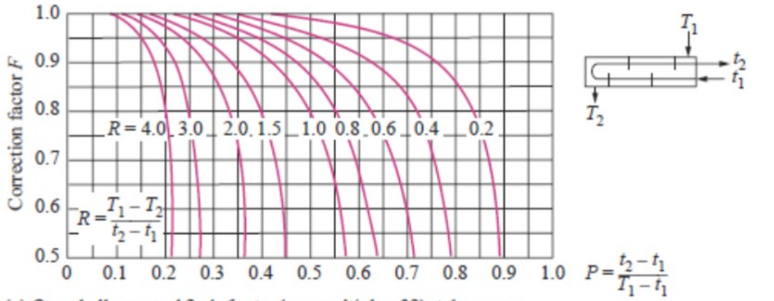
- Çengel, Y.A. (2002). *Heat Transfer: A Practical Approach*, 2nd Edition, McGraw-Hill: Chapter 13
- Holman, J.P. (2010). *Heat Transfer*, 10th Edition, McGraw-Hill: Chapter 10
- Incropera, F.P., DeWitt, D.P., Bergman, T.L., Lavine, A.S. (2006). *Introduction to Heat Transfer*, 6th Edition, John Wiley & Sons: Chapter 11
- Kakaç, S., Liu, H., Pramuanjaroenkij, A. (2012). *Heat Exchangers: Selection, Rating, and Thermal Design*, 3rd Edition, Taylor & Francis: Chapters 1 – 2
- Karwa, R. (2020). *Heat and Mass Transfer*, 2nd Edition, Springer: Chapter 14
- Shah, R.K., Sekulić, D.P. (2003). *Fundamentals of Heat Exchanger Design*, John Wiley & Sons: Chapter 1
- Theodore, L. (2011). *Essential Engineering Calculations Series: Heat Transfer Applications for the Practicing Engineer*, John Wiley & Sons: Chapter 14

A Classification of heat exchangers

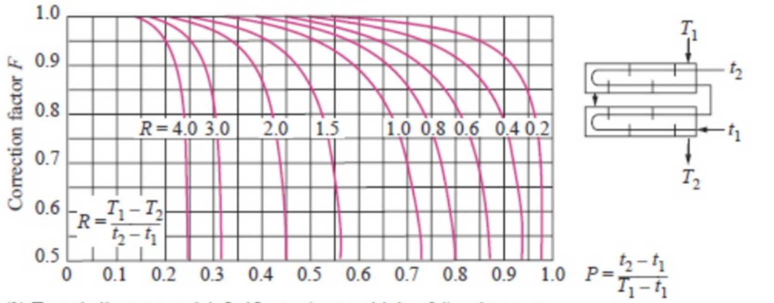


(from Shah & Sekulić, 2003, Figure 1.1, p2)

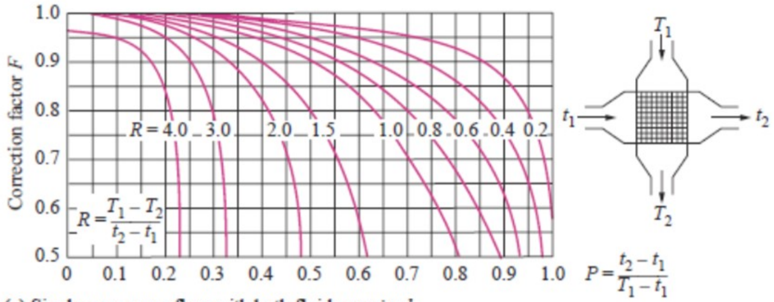
B LMTD correction factor charts



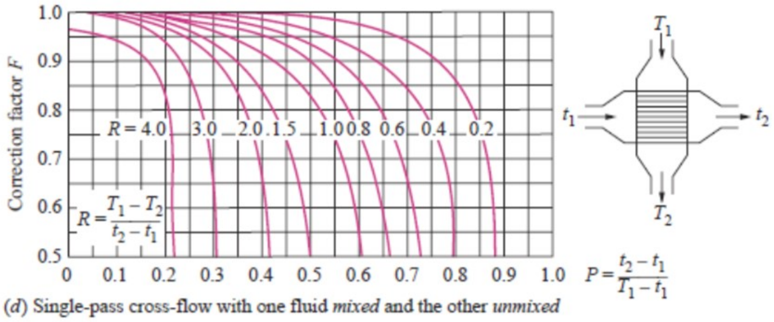
(*a*) One-shell pass and 2, 4, 6, etc. (any multiple of 2), tube passes



(*b*) Two-shell passes and 4, 8, 12, etc. (any multiple of 4), tube passes



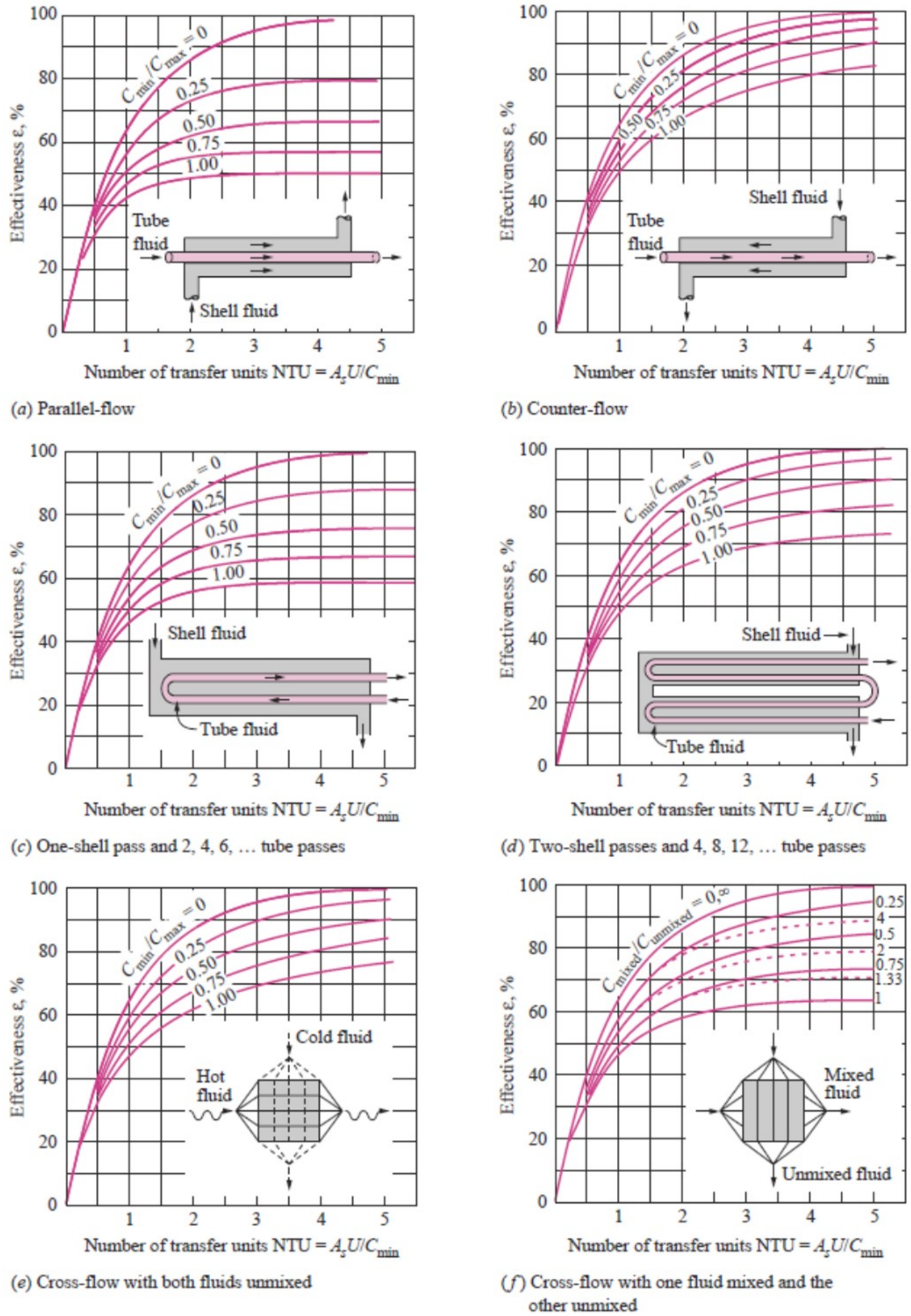
(*c*) Single-pass cross-flow with both fluids *unmixed*



(*d*) Single-pass cross-flow with one fluid *mixed* and the other *unmixed*

(from Çengel, 2002, Figure 13-18, p684)

C Effectiveness-NTU charts



(from Çengel, 2002, Figure 13-26, p695)

D Effectiveness relations for heat exchangers

Effectiveness relations for heat exchangers: NTU $= UA_s/C_{min}$ and $c = C_{min}/C_{max} = (\dot{m}C_p)_{min}/(\dot{m}C_p)_{max}$ (Kays and London, Ref. 5.)

Heat exchanger type	Effectiveness relation
1 *Double pipe:*	
Parallel-flow	$\varepsilon = \dfrac{1 - \exp[-\mathrm{NTU}(1+c)]}{1+c}$
Counter-flow	$\varepsilon = \dfrac{1 - \exp[-\mathrm{NTU}(1-c)]}{1 - c\exp[-\mathrm{NTU}(1-c)]}$
2 *Shell and tube:*	
One-shell pass 2, 4, . . . tube passes	$\varepsilon = 2\left\{1 + c + \sqrt{1+c^2}\,\dfrac{1+\exp[-\mathrm{NTU}\sqrt{1+c^2}]}{1-\exp[-\mathrm{NTU}\sqrt{1+c^2}]}\right\}^{-1}$
3 *Cross-flow (single-pass)*	
Both fluids unmixed	$\varepsilon = 1 - \exp\left\{\dfrac{\mathrm{NTU}^{0.22}}{c}[\exp(-c\ \mathrm{NTU}^{0.78}) - 1]\right\}$
C_{max} mixed, C_{min} unmixed	$\varepsilon = \dfrac{1}{c}(1 - \exp\{1 - c[1 - \exp(-\mathrm{NTU})]\})$
C_{min} mixed, C_{max} unmixed	$\varepsilon = 1 - \exp\left\{-\dfrac{1}{c}[1 - \exp(-c\,\mathrm{NTU})]\right\}$
4 *All heat exchangers with c = 0*	$\varepsilon = 1 - \exp(-\mathrm{NTU})$

(from Çengel, 2002, Table 13-4, p684)